MAT2705-04/05 22S Test 1 Answers

① $\frac{dT}{dt} = -k(T-A)$, $T(0)=T_0$, $A=72$, $T_0=212$

a) $\frac{dT}{dt} = -k(T-72)$

$\int \frac{dT}{T-72} = \int -k\,dt$

$\ln|T-72| = -kt + C_1$

$e^{\ln|T-72|} = e^{-kt+C_1}$

$|T-72| = e^{C_1}e^{-kt}$

$T-72 = \pm e^{C_1}e^{-kt}$, C

$\boxed{T = 72 + Ce^{-kt}}$

b) $212 = T(0) = 72 + Ce^0$

$212-72 = C \rightarrow C = 140$

$\boxed{T = 72 + 140e^{-kt}}$

c) $107 = T(20) = 72 + 140e^{-20k}$

$\frac{107-72}{140} = e^{-20k}$

$\frac{35}{140} = \frac{1}{4}$

$4 = e^{20k}$

$20k = \ln 4$

$k = \frac{\ln 4}{20}$

$T = 72 + 140e^{-\frac{\ln 4}{20}t}$

$(e^{\ln 4})^{-\frac{t}{20}}$

$\boxed{T = 72 + 140\,4^{-t/20}}$

$(2^2)^{-t/20} = 2^{-t/10}$

half-life $= 10$

$\frac{\theta}{70} = \frac{10}{140} = 4^{-t/20} \rightarrow$

d) $90 = 72 + 140\,4^{-t/20}$

$4^{t/20} = \frac{70}{9}$

$\frac{t}{20}\ln 4 = \ln\frac{70}{9}$

$t = \frac{\ln 70/9}{\ln 4}\,20 \approx 29.5936$

≈ 29 min $+ \underbrace{0.5936(60)}_{35.6}$ sec

The temperature should reach 90° at about

29 min 36 sec.

② $xy' - 2y = 6x^3\cos(2x)$

$\left[y' - \frac{2}{x}y = 6x^2\cos(2x)\right]$

$\int -\frac{2}{x}dx = -2\ln|x|$

$e^{-2\ln|x|} = |x|^{-2} = x^{-2}$

$x^{-2}\left(y' - \frac{2}{x}y\right) = x^{-2}(6x^2\cos 2x)$

$\frac{d}{dx}(yx^{-2}) = 6\cos 2x$

$yx^{-2} = \int 6\cos 2x\,dx$

$= \frac{6}{2}\sin 2x + C$, $\frac{6}{2} = 3$

$\boxed{y = x^2(3\sin 2x + C)}$

$= 3x^2\sin 2x + Cx^2$

b) $3 = y(1) = 3(1)^2\sin 2 + C(1)^2$

$C = 3 - 3\sin 2$

$\boxed{y = 3x^2\sin 2x + (3-3\sin 2)x^2}$

$= 3x^2(\sin 2x - \sin 2 + 1)$

$y' = 6x(\sin 2x - \sin 2 + 1) + 3x^2(2\cos 2x)$

$xy' - 2y = 6x^3\cos 2x$

$x[6x(\sin 2x - \sin 2 + 1) + 3x^2(2\cos 2x)] - 2(3x^2)(\sin 2x - \sin 2 + 1) = 6x^3\cos 2x$

$6x^2(\sin 2x - \sin 2 + 1) + 6x^3\cos 2x - 6x^2(\sin 2x - \sin 2 + 1) = 6x^3\cos 2x$

$6x^3\cos 2x = 6x^3\cos 2x$ ✓