MAT2500-05 22F Quiz 9 Print Name (Last, First) _____________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them.

Consider the circle in the plane described by the polar equation $r = 2\cos(\theta)$. Use Maple to evaluate the required iterated integrals below (since they require integrating powers of trig functions).

a) Make a rough sketch of this region with equally spaced linear cross sections in the radial direction from the origin which sweep out the region. What is the range of the angular coordinate for which $r \geq 0$? Label a typical such linear cross section by its starting and stopping values of r, with an arrow midway in the outward direction.

b) Using your diagram, write down the limits of integration that correspond to part a) for any integral

$$\iint_R f\, dA = \int_{\theta_1}^{\theta_2} \int_{r_1}^{r_2} f\, r\, dr\, d\theta$$

fraction division: $\dfrac{\frac{64}{15}}{32/9} = \dfrac{\cancel{64}^{2} \cdot 9}{15 \cdot \cancel{32}} = \dfrac{2 \cdot 9}{15} = \dfrac{2 \cdot 3 \cdot 3}{3 \cdot 5} = \dfrac{6}{5}$

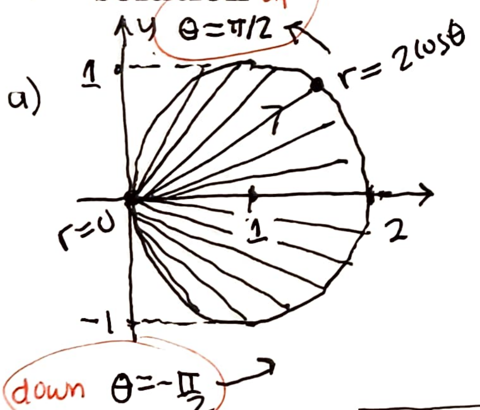
c) Evaluate the integrals of the three scalars $1, x, y = 1, r\cos(\theta), r\sin(\theta)$ to yield the area A and its two moments Ay, Ax from which, calculate the centroid coordinates $\langle x_c, y_c\rangle = \dfrac{\langle A_y, A_x\rangle}{A}$, which should agree with your knowledge of the centroid of a circle.

d) For a mass density $\rho = k\, r$ proportional to the distance from the origin, recalculate the mass M and moments My, Mx from which, calculate the center of mass coordinates $\langle x_{cm}, y_{cm}\rangle = \dfrac{\langle M_y, M_x\rangle}{M}$. Does this move from the centroid in the right direction? Explain.

e) Make a rough sketch of the circle locating the two points just calculated, including stating the decimal values of their coordinates. Be sure to include adequate tickmarks and labels.

▸ **solution**

a)



$r = 0 .. 2\cos\theta$ while $\boxed{\theta = -\frac{\pi}{2} .. \frac{\pi}{2}}$

note ∵ $2\cos\theta \geq 0$ on this interval

b) $\iint_R f\, dA = \boxed{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{2\cos\theta} f\, r\, dr\, d\theta}$

c) $\langle A, A_y, A_x\rangle = \iint_R \langle 1, r\cos\theta, r\sin\theta\rangle$

$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{2\cos\theta} \underbrace{\langle 1, r\cos\theta, r\sin\theta\rangle\, r}_{= \langle r, r^2\cos\theta, r^2\sin\theta\rangle}\, dr\, d\theta$

$\underset{\text{Maple}}{=} \langle \pi, \pi, 0\rangle$

so $\langle x_c, y_c\rangle = \dfrac{\langle A_y, A_x\rangle}{A} = \dfrac{\langle \pi, 0\rangle}{\pi}$

$= \boxed{\langle 1, 0\rangle}$ center of the circle

d) $\langle M, M_y, M_x\rangle = \iint_R kr\langle 1, r\cos\theta, r\sin\theta\rangle\, r\, dr\, d\theta$

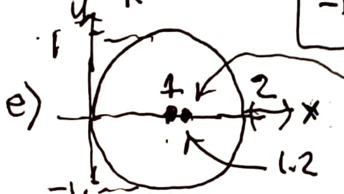
$= k\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{2\cos\theta} \underbrace{\langle r, r^2\cos\theta, r^2\sin\theta\rangle\, r}_{\langle r^3, r^3\cos\theta, r^3\sin\theta\rangle}\, dr\, d\theta$

$\underset{\text{Maple}}{=} k\left\langle \frac{32}{9}, \frac{64}{15}, 0\right\rangle$

so $\langle x_{cm}, y_{cm}\rangle = \dfrac{\langle M_y, M_x\rangle}{M} = \dfrac{\langle \frac{64}{15}, 0\rangle}{32/9}$

$= \boxed{\langle \frac{6}{5}, 0\rangle = \langle 1.2, 0\rangle}$ shifts right since more mass away from origin

e)



(x_{cm}, y_{cm})