MAT2500-05 22F Quiz 8 Print Name (Last, First) _____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them.

1. Find the volume under the surface $z = 1 + x^2\ y^2$ above the region enclosed by $x = y^2$ and $x = 4$. First set up the two iterated integrals representing this volume and in each case let Maple evaluate the double integral. Follow these steps:

a) Integrate first in the horizontal direction.

b) Integrate first in the vertical direction.

In each case accompany your work with a new iteration diagram to justify your interation, a diagram shaded by equally spaced linear cross-sections and a typical one with bullet point endpoints labeled by the equation of the starting and stopping values of the integration variable for the inner integral and with an arrowhead midway indicating the variable's increasing direction.

c) Use Maple to evaluate each such integral exactly. Do they agree as they should?

2. a) "Deconstruct" the integral $\int_0^1 \int_{\sqrt{y}}^1 x\cos(y)\,dx\,dy$ by creating a diagram as above that explains the limits of integration in the current order.

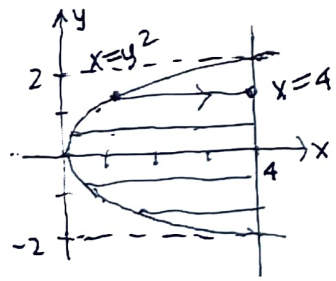
b) Create a new diagram explaining the limits for the reversed order of integration.

c) Write down the new iteration of this integral in that reversed order and evaluate it step by step by hand.

d) Check with Maple that the two iterations have the same value. [The first integration requires an integration by parts (bad!), the second only a u-substitution (good!).]

▶ solution

(i) a)

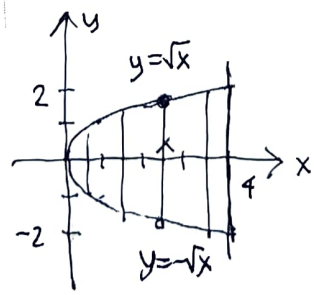


$x = y^2 .. 4$ while $y = -2..2$

$$V = \int_{-2}^{2}\int_{y^2}^{4} (1 + x^2y^2)\,dx\,dy$$

c) $= \frac{2336}{27} \approx 86.52$

$x = y^2 \rightarrow y = \pm\sqrt{x}$



$y = -\sqrt{x} .. \sqrt{x}$ while $x = 0..4$

$$V = \int_0^4 \int_{-\sqrt{x}}^{\sqrt{x}} (1 + x^2y^2)\,dy\,dx$$

$= \frac{2336}{27}$ ✓ they do agree!

$1 + x^2y^2$

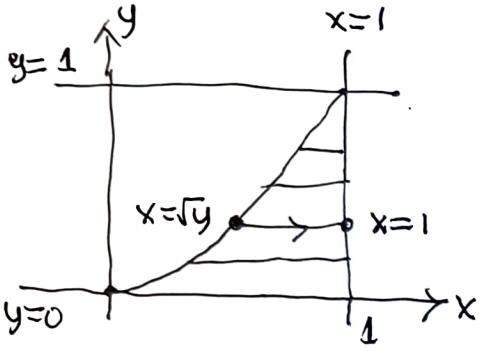
need space or asterisk for multiplication or Maple ignores this term and evaluates with integrand 1.

MAT 2500-05 22F Quiz 8

a) $\int_{y=0}^{y=1} \int_{x=\sqrt{y}}^{x=1} x\cos y \, dx \, dy$

$\updownarrow$ $y = x^2$ ↘ invert for later

boundary curves



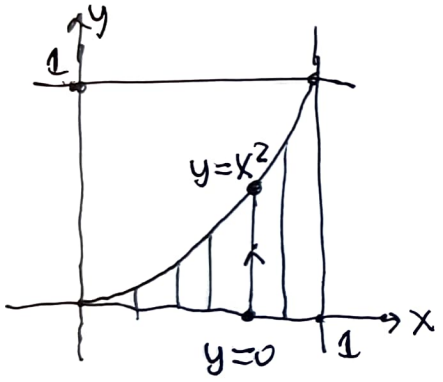
$x = \sqrt{y} \,..\, 1$ horizontal cross-sections

↑ left ↑ right endpoints

picks out which of two subregions in unit square is region of integration

while $y = 0 \,..\, 1$

b)



$y = 0 \,..\, x^2$ while $x = 0 \,..\, 1$

c) $\boxed{\int_0^1 \int_0^{x^2} x\cos y \, dy \, dx} = \int_0^1 \left(x \sin y \Big|_{y=0}^{y=x^2} \right) dx$

$= \int_0^1 x(\sin x^2 - \sin 0) \, dx = \int_0^1 \underbrace{x \sin x^2 \, dx}_{\frac{dy}{2}} = \int_{x=0}^{x=1} \sin u \, \frac{dy}{2}$ (u = x^2)

$u = x^2$

$du = 2x\,dx$

$x\,dx = \frac{1}{2} du$

$= -\frac{1}{2}\cos u \Big|_{x=0}^{x=1} = -\frac{1}{2}\cos x^2 \Big|_0^1 = -\frac{1}{2}\cos 1 + \frac{1}{2} = \boxed{\frac{1}{2}(1 - \cos 1)}$

d) Maple gives this same result for <u>both</u> integrals.