MAT2500-05 22f Quiz 7 Print Name (Last, First) ____________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, IDENTIFYING expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation if appropriate). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them.

two obvious typos: my fault!

1. Let $F(x,y,z) = y^2 e^{xyz}$, $P(0,1,-1)$, $v = \frac{1}{13}\langle 3,4,12\rangle$

F! → $f \neq F$ math is case sensitive!

a) Find the gradient of f and evaluate it at the point F.

P! oops, another typo but context makes it clear

b) Evaluate its magnitude and unit vector direction u at this point.

c) Evaluate the directional derivative of F along v at this point.

d) Write the equation of the level surface through this point.

e) Write the equation of the tangent plane to the level surface through this point and reduce it to the simple form $a\,x + b\,y + c\,z = d$.

f) Write the equation for the normal line through this point using u for the orientation vector so that it is arclength parametrized, letting s be the parameter in this equation.

g) Find the numerical value of s for which $F(\vec{r}(s)) = F(0,1,-1)$. Search in the interval $s=-3..0$. There are two disjoint surfaces in this level surface (because of the factor y^2, when solving the level surface equation for y), and the normal line from one of them hits the other one.

h) **Optional**.

Do an implicitplot3d of the F for $x=-2..2$, $y=-2..2$, $z=-2..2$ and copy and paste the spacecurve of the normal line for $s=-3..1$ to see how parts e) and f) work. The segment $s=0..1$ reproduces the gradient vector. And then copy and paste an implicitplot3d of the tangent plane onto this as well. See the Maple answer key later to view the result.

solution

a) $F(x,y,z) = y^2 e^{xyz}$

arrows over vector symbols!

$\vec{\nabla}F(xyz) = \langle y^2 \cdot yz\, e^{xyz},\ 2y e^{xyz} + y^2 \cdot xz\, e^{xyz},\ y^2(xy)\, e^{xyz}\rangle$

$= \boxed{e^{xyz}\langle y^3 z,\ 2y + y^2 xz,\ xy^3\rangle = y e^{xyz}\langle y^2 z,\ 2 + xyz,\ xy^2\rangle}$

"its" refers to the gradient; the gradient is interpreted thru its magnitude & direction

$\vec{\nabla}F(0,1,-1) = 1e^0 \langle 1(-1), 2+0, 0\cdot 1\rangle = \boxed{\langle -1,2,0\rangle}$

b) $|\vec{\nabla}F(0,1,-1)| = \sqrt{1+4} = \boxed{\sqrt{5}}$ $\quad \hat{u} = \frac{\vec{\nabla}F(0,1,-1)}{|\vec{\nabla}F(0,1,-1)|} = \boxed{\frac{\langle -1,2,0\rangle}{\sqrt{5}}}$

c) $D_{\hat{v}}F(0,1,-1) = \hat{v}\cdot\vec{\nabla}F(0,1,-1) = \frac{1}{13}\langle 3,4,12\rangle \cdot \langle -1,2,0\rangle = \frac{1}{13}(-3+8) = \boxed{\frac{5}{13}}$

d) $F(0,1,-1) = 1e^0 = 1$ so $\boxed{y^2 e^{xyz} = 1}$

Note $y^2 = e^{-xyz}$ 2 branches of solution: $y>0$, $y<0$. Maple can solve this!

e) $\vec{n} = \langle -1,2,0\rangle$, $\vec{r}_0 = \langle 0,1,-1\rangle$

$0 = \vec{n}\cdot(\vec{r}-\vec{r}_0) = \langle -1,2,0\rangle \cdot \langle x-0, y-1, z+1\rangle = -x + 2(y-1) + 0 = -x+2y-2$

$\boxed{-x+2y=2}$ vertical plane

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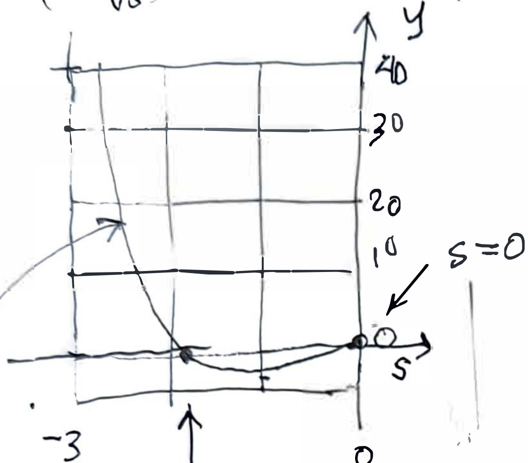
f) $$\boxed{\vec r = r_0 + s\hat u = \langle 0,1,-1\rangle + \frac{s}{\sqrt5}\langle -1,2,0\rangle = \left\langle -\frac{s}{\sqrt5},\ 1+\frac{2s}{\sqrt5},\ -1\right\rangle = \langle x,y,z\rangle}$$

g) $$1 = F(\vec r(s)) = \left(1+\frac{2s}{\sqrt5}\right)^2 e^{-\frac{s}{\sqrt5}\left(1+\frac{2s}{\sqrt5}\right)(-1)}$$

$$= \left(1+\frac{2s}{\sqrt5}\right)^2 e^{\frac{s}{\sqrt5}\left(1+\frac{2s}{\sqrt5}\right)} \xrightarrow{\text{Maple}} \boxed{s \approx -1.937}$$



always plot a function when trying to numerically solve for its roots

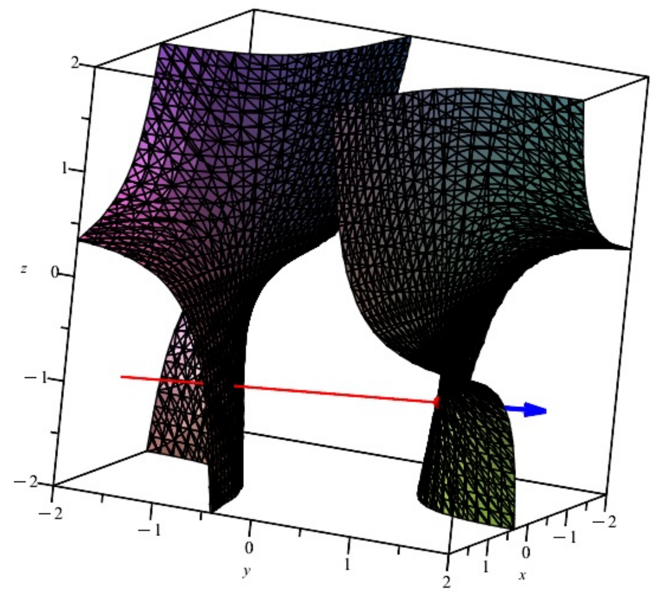
graph of:

$$y = \left(1+\frac{2s}{\sqrt5}\right)^2 e^{\frac{s}{\sqrt5}\left(1+\frac{2s}{\sqrt5}\right)} - 1 \underbrace{= 0}_{\text{roots desired.}}$$

If you dont use unit vector:

$$\vec r(t) = r_0 + t\vec u$$
$$= \langle 0,1,-1\rangle + t\langle -1,2,0\rangle$$
$$= \langle 0,1,-1\rangle + s\frac{\langle -1,2,0\rangle}{\sqrt5}$$

$$\rightarrow\ t = \frac{s}{\sqrt5} \approx \frac{-1.937}{\sqrt5}$$
$$\approx 0.865$$



normal line on right sheet at P hits left sheet