MAT2500-05 22f Quiz 6 Print Name (Last, First) ______________________________

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, IDENTIFYING expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation if appropriate). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them.

1. Suppose we compare the area of the European letter size A4 dimensions of 21.0x29.7 cm to our American standard letter size 8.5x11 in. [Recall 1in = 2.54 cm.]

a) Letting $[x, y] = [8.5, 11.0]$ evaluate the differentials of the dimensions of the A4 size paper compared with our letter size dimensions, and use the differential approximation to evaluate the approximate change in area and its fractional (and percentage) change.

b) Then compare this approximate change in area with the exact such change in the area: $dA - \Delta A$.

c) Use the linear approximation to the ratio of vertical to horizontal dimension to evaluate the approximate ratio of the European A4 dimensions, and compare to the actual ratio (evaluate the difference over the actual ratio).

2. Error Analysis.

The Great Pyramid of King Khufu was built of limestone in Egypt over a 20-year time period from 2580 BC to 2560 BC. Its base is a square with side length 756 ft and its height when built was 481 ft.

The volume formula is $V = \frac{1}{3}s^2 h$

If these measurements are only known to be accurate to the nearest foot, what is the maximum absolute error in the computed volume using the differential approximation? How does that compare to the exact change if we evaluate the exact volume at the upper end of the error bar for each dimension? [Hint: which is the interval around each measurement that allows them to be "to the nearest foot"?]

▶ **solution**

natural for us Americans to use inches!

but worked in cm: $[x,y] = [21.59, 27.94]$
$[x+dx, y+dy] = [21.0, 29.7]$
$[dx, dy] = [-0.59, 1.76]$
$dA = 21.5138$
$\frac{dA}{A} = 0.0357$
$A + \Delta A = 21.0 \cdot 29.7 = 623.7$
$\Delta A = 20.4754$
$dA - \Delta A = 1.0384$

1. a) $A = xy$, $dA = y\,dx + x\,dy$

$[x,y] = [8.5, 11.0] \rightarrow A = 8.5(11.0) = 93.50$

$[x+dx, y+dy] = \frac{1}{2.54}[21.0, 29.7] = [8.2677, 11.6929]$

$\rightarrow [dx, dy] = [-0.2323, 0.6929]$

$dA = 11.0(-0.2323) + 8.5(0.6929) = \boxed{3.3346}$

$\frac{dA}{A} = \frac{3.3346}{93.50} = 0.0357$ Eur paper has about $\boxed{3\frac{1}{2}\%}$ more area.

b) But $A + \Delta A = (x+dx)(y+dy) = 96.674$ so $\Delta A = 3.1737$

$dA - \Delta A = 0.1610$ dA is a little high

MAT2500-05 22F Quiz 6

(1) c) $R(x,y) = \frac{y}{x} = x^{-1}y$ $\quad R(8.5, 11.0) = 1.2941$

$R(8.2677, 11.6929) = 1.4143$ independent of units!

$\frac{\partial R}{\partial x} = -x^{-2}y = -\frac{y}{x^2}$ $\quad (x_0, y_0) = (8.5, 11.0)$

$\frac{\partial R}{\partial y} = \frac{1}{x}$

$$L(x,y) = R(x_0,y_0) + \frac{\partial R}{\partial x}(x_0 y_0)(x-x_0) + \frac{\partial R}{\partial y}(y-y_0)$$

$$= 1.2941 - 0.1522(x-8.5) + 0.1176(y-11.0)$$

$$L(8.2677, 11.6929) = 1.2941 - 0.1522(-0.2323) + 0.1176(0.6929) = 1.4110 \text{ (maple)}$$

$L(8.2677, 11.6929) = 1.4110$ (maple)

$L(x+\Delta x, y+\Delta y) - R(x+\Delta x, y+\Delta y) = 1.4110 - 1.4143 = \boxed{-0.0033}$ $\longrightarrow$ $\frac{\Delta L}{R(x,y)} = \frac{-0.0033}{1.4143} = -0.0023$

The linear approximation is a bit low. (about 0.2% too low)

(2) $V = \frac{1}{3}s^2h$ $\quad \frac{\partial V}{\partial s} = \frac{2}{3}sh$, $\quad \frac{\partial V}{\partial h} = \frac{1}{3}s^2$

$s = 756 \quad ds = 0.5$ $\qquad V = \ldots = 9.164 \cdot 10^7$

$h = 481 \quad dh = 0.5$ $\qquad V|_{s+ds,h+dh} = 9.185 \cdot 10^7$ $\Big] \to \Delta V$ difference

$$dV = \frac{\partial V}{\partial s}ds + \frac{\partial V}{\partial h}dh = \frac{2}{3}shds + \frac{1}{3}s^2dh = \frac{s}{3}(2hds + sdh)$$

$$|dV| \le \frac{s}{3}(2h\underbrace{|ds|}_{\le 0.5} + s\underbrace{|dh|}_{\le 0.5}) \le \frac{0.5s}{3}(2h+s)$$

$= \boxed{\begin{array}{l} 216{,}468 \leftarrow \text{a bit low compared to} \\ 216\ 634 \leftarrow \text{exact max change} \end{array}}$

$\Delta V =$

$\Delta V - |dV|_{\text{estimate max}} = 166.13$ pretty small difference.