MAT2500-05 22f Quiz 5 Print Name (Last, First) ______________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, IDENTIFYING expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation if appropriate). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them.

1. $f(x,y)=\ln\left(\dfrac{10+\sqrt{x^2+y^2}}{10-\sqrt{x^2+y^2}}\right)$. Evaluate $f_x(x,y)$ and its value at $(3,4)$, identifying each by their proper symbols. [Check your work with Maple! Also this can be done in two ways: ln (quotient) or difference of ln's. Combine fractions afterwards whichever route you choose.]

2. The wave heights h in open sea depend on the speed v of the wind and the length of time t that the wind has been blowing at that speed. Values of the function $h=f(v,t)$ are recorded in feet in the table.

a) What is the value of $f(50, 30)$? What is its meaning in words (a complete sentence)?
b) For what value of t is $f(50, t) = 29$? Formulate this question in words.
c) What is the meaning of the function $h = f(v, 30)$?
d) Evaluate the tabular partial derivative $\dfrac{\partial f}{\partial v}(50, 30)$.
e) If the wind speed increases by 1 knot at these values, what do you expect the change in the height to be, and what would the new height be?

Duration (hours)

Wind speed (knots) v \ t	5	10	15	20	30	40	50
10	2	2	2	2	2	2	2
15	4	4	5	5	5	5	5
20	5	7	8	8	9	9	9
30	9	13	16	17	18	19	19
40	14	21	25	28	31	33	33
50	19	29	36	40	45	48	50
60	24	37	47	54	62	67	69

▶ solution

① $\dfrac{\partial f}{\partial x}=\dfrac{\partial}{\partial x}\ln\left(\dfrac{10+\sqrt{x^2+y^2}}{10-\sqrt{x^2+y^2}}\right)=\dfrac{1}{\frac{10+\sqrt{x^2+y^2}}{10-\sqrt{x^2+y^2}}}\cdot\dfrac{(10-\sqrt{x^2+y^2})(\frac12(x^2+y^2)^{-1/2}(2x))-(10+\sqrt{x^2+y^2})(-\frac12(x^2+y^2)^{-1/2}(2x))}{(10-\sqrt{x^2+y^2})^2}$

$=\dfrac{x}{\sqrt{x^2+y^2}\,(10+\sqrt{x^2+y^2})(10-\sqrt{x^2+y^2})}\left(10-\sqrt{x^2+y^2}+10+\sqrt{x^2+y^2}\right)$

$=\boxed{\dfrac{20x}{\sqrt{x^2+y^2}\,(100-(x^2+y^2))}}$

OR

$10-\sqrt{x^2+y^2}+10+\sqrt{x^2+y^2}$

$\dfrac{\partial f}{\partial x}=\dfrac{\partial}{\partial x}\left(\ln(10+\sqrt{x^2+y^2})-\ln(10-\sqrt{x^2+y^2})\right)$

$=\dfrac{\frac12(x^2+y^2)^{-1/2}(2x)}{10+\sqrt{x^2+y^2}}-\dfrac{(-\frac12)(x^2+y^2)^{-1/2}(2x)}{10-\sqrt{x^2+y^2}}=\dfrac{x}{\sqrt{x^2+y^2}}\left(\dfrac{1}{10+\sqrt{x^2+y^2}}+\dfrac{1}{10-\sqrt{x^2+y^2}}\right)$

$=\boxed{\dfrac{20x}{\sqrt{x^2+y^2}}\dfrac{1}{(10+\sqrt{x^2+y^2})(10-\sqrt{x^2+y^2})}=\dfrac{20x}{\sqrt{x^2+y^2}\,(100-(x^2+y^2))}}$

either acceptable but $a^2-b^2=(a-b)(a+b)$ should be recognized!

$f_x(3,4)=\dfrac{20(3)}{5(100-25)}=\dfrac{20\cdot3}{5\cdot75}=\dfrac{20}{25\cdot5}=\dfrac{4}{5\cdot5}=\boxed{\dfrac{4}{25}}$

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(2) a) $f(50, 30) = 45$ — row v, col t

When the wind has been blowing for 30 hours at 50 knots, the wave height is 45 feet.

b) $f(50, t) = 29$? $\rightarrow$ $\boxed{t=10}$

How long must the wind blow at 50 knots for the wave height to be 29 feet?

c) $h = f(v, 30)$ This gives the wave height as a function of the wind speed after 30 hours of blowing.

d) $\frac{\partial f}{\partial v}(50,30)$: (v, t)

v \ t	30
40	31
50	45
60	62

$\Delta v = 10$

$\Delta h = 45-31 = 14$ $\quad \frac{\Delta h}{\Delta v} = \frac{14}{10} = 1.4$

$\Delta h = 62-45 = 17$ $\quad \frac{\Delta h}{\Delta v} = \frac{17}{10} = 1.7$

$\frac{1}{2}\left(\left(\frac{\Delta h}{\Delta v}\right)_- + \left(\frac{\Delta h}{\Delta v}\right)_+\right) = \frac{1}{2}(1.4+1.7) = 1.55$

$$\boxed{\frac{\partial f}{\partial v}(50,30) = 1.55}$$

e) $\Delta v = 1$:

$\Delta h \approx \frac{\partial f}{\partial v}(50,30)\,\Delta v = \boxed{1.55}$

$h + \Delta h = 45 + 1.55 = \boxed{46.55}$