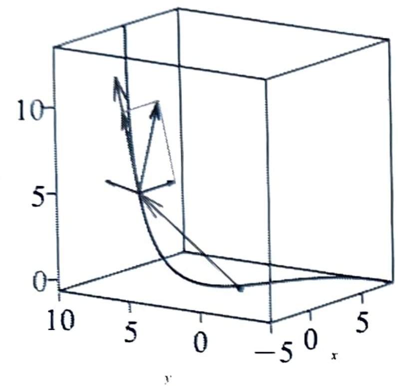
MAT2500-05 22F Quiz 4 Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, IDENTIFYING expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them.



Given the vector-valued function $\vec{r}(t) = \left\langle t^2 - 2t, 1 + 3t, \frac{1}{3}t^3 + \frac{1}{2}t^2 \right\rangle$ for the domain $t = -1..3$ (no credit for unidentified expressions):

a) Evaluate $\vec{r}\,'(t)$, $\vec{r}\,''(t)$, $|\vec{r}\,'(t)|$, $\hat{T}(t)$ and remember to simplify your results.

b) Evaluate $\vec{r}(2)$, $\vec{r}\,'(2)$, $\vec{r}\,''(2)$, $\hat{T}(2)$ and remember to simplify your results.

c) Evaluate the exact angle θ in radians between $\vec{r}\,'(2)$ and $\vec{r}\,''(2)$ and a single decimal place approximation in degrees. Does it seem compatible with the figure, which shows the position vector from the origin, the first and second derivatives and the latter's projections with respect to the unit tangent?

d) Write the simplified equation for the plane through $\vec{r}(2)$ containing the first and second derivatives there as shown in the figure. What is its distance of this plane from the origin to 3 decimal places? [How do we find the distance between a point and a plane?]

e) Evaluate the vector projections $\vec{a}_{\parallel}$ and $\vec{a}_{\perp}$ of $\vec{a} = \vec{r}\,''(2)$ along $\vec{r}\,'(2)$.

▶ solution

a) $\vec{r} = \langle t^2-2t, 1+3t, \frac{1}{3}t^3+\frac{1}{2}t^2\rangle$

$\vec{r}\,' = \langle 2t-2, 3, t^2+t\rangle$

$\vec{r}\,'' = \langle 2, 0, 2t+1\rangle$

$|\vec{r}\,'| = \sqrt{(2t-2)^2+3^2+(t^2+t)^2}$

$4t^2-8t+4$

$+9$

$+t^2+2t^3+t^4$

$= \sqrt{5t^2+2t^3+t^4-8t+13}$

$\hat{T} = \dfrac{\langle 2t-2, 3, t^2+t\rangle}{\sqrt{t^4+2t^3+5t^2-8t+13}}$

b) $\vec{r}(2) = \langle 2^2-4, 1+6, \frac{1}{3}8+\frac{1}{2}4\rangle$

$= \langle 0, 7, 14/3\rangle$

$\vec{r}\,'(2) = \langle 2(2)-2, 3, 4+2\rangle = \langle 2,3,6\rangle$

$|\vec{r}\,'(2)| = \sqrt{4+9+36} = 7$

$\hat{T}(2) = \frac{1}{7}\langle 2,3,6\rangle$

c) $\vec{r}\,''(2) = \cancel{\sqrt{7^2+14^2/9}} = \cancel{\sqrt{9\cdot 7^2+14^2}/3}$

$= \cancel{\sqrt{4+(4t^2+4t+1)}} = \cancel{\sqrt{5+4t+4t^2}}$ TV distraction

$= \langle 2,0,5\rangle$

$\hat{r}''(2) = \dfrac{\langle 2,0,5\rangle}{\sqrt{29}}$

c) continued:

$\cos\theta = \hat{T}(2)\cdot\hat{r}''(2) = \frac{1}{7}\langle 2,3,6\rangle\cdot\frac{1}{\sqrt{29}}\langle 2,0,5\rangle = \frac{1}{7\sqrt{29}}(4+30) = \frac{34}{7\sqrt{29}}$

$\theta = \arccos\left(\frac{34}{7\sqrt{20}}\right) \approx \boxed{25.6^\circ}$ Looks like in the diagram, so yes!

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d) $0 = \vec{n}\cdot(\vec{r}-\vec{r}_0)$:

$r_0 = \vec{r}(2) = \langle 0, 7, 14/3\rangle = \frac{1}{3}\langle 0, 21, 14\rangle = \frac{7}{3}\langle 0, 3, 2\rangle$

$\vec{r}\,'(2)\times\vec{r}\,''(2) = \langle 2, 3, 6\rangle \times \langle 2, 0, 5\rangle$

$\underset{\text{Maple}}{=} \langle 15, 2, -6\rangle = \vec{n}$

$0 = \langle 15, 2, -6\rangle \cdot \langle x-0, y-7, z-14/3\rangle$

$= 15(x) + 2(y-7) - 6(z-14/3)$

$= 15x + 2y - 6z \underbrace{-14+28}_{14} \rightarrow \boxed{15x+2y-6z=-14}$

$\hat{n}\times\vec{r}_0 = \frac{\langle 15, 2, -6\rangle}{\sqrt{15^2+4+36}} \times \frac{7}{3}\langle 0, 3, 2\rangle$

$= \frac{7}{3\sqrt{265}}(6-12) = -\frac{14}{\sqrt{265}} \rightarrow d = |\hat{n}\times\vec{r}_0| = \boxed{\frac{14}{\sqrt{265}} \approx 0.8600}$

e) $\vec{a} = \vec{r}\,''(2) = \langle 2, 0, 5\rangle$

$a_{\parallel} = \hat{T}(2)\cdot\vec{a} = \frac{1}{7}\langle 2, 3, 6\rangle\cdot\langle 2, 0, 5\rangle = \frac{1}{7}(4+30) = \frac{34}{7}$

$\vec{a}_{\parallel} = a_{\parallel}\hat{T}(2) = \frac{1}{7}\cdot\frac{34}{7}\langle 2, 3, 6\rangle = \boxed{\frac{34}{49}\langle 2, 3, 6\rangle} = \boxed{\frac{\langle 68, 102, 204\rangle}{49}}$

$\vec{a}_{\perp} = \vec{a} - \vec{a}_{\parallel} = \langle 2, 0, 5\rangle - \frac{\langle 68, 102, 204\rangle}{49}$

$\underset{\text{Maple}}{=} \boxed{\frac{\langle 30, -102, 41\rangle}{49}}$