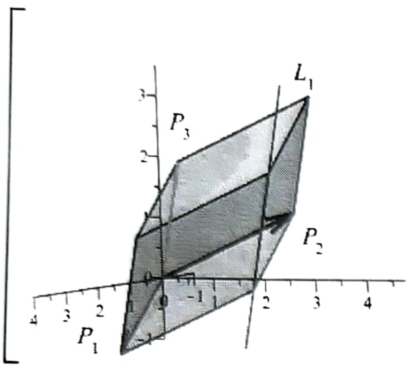
MAT2500-05 22F Quiz 3 Print Name (Last, First) ______________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not subsitute for them.

Given three points $P_1(3, 1, -1)$, $P_2(-1, 2, 1)$, $P_3(1, 1, 2)$ and the parallelopiped formed from their three position vectors $\vec{r}_1, \vec{r}_2, \vec{r}_3$. [Note $\vec{r}_1$ comes forward to the front face of this object, with the parallelogram of $\vec{r}_2, \vec{r}_3$ equaling the back face.]



a) Write the parametrized equations of the line L_1 through the right edge of the front face $\mathscr{P}_{frontface}$ as shown. [What is the simplest position vector of a point on this line? What is the orientation of the line?] Where does this line intersect the $z = 0$ plane?

b) Find a normal vector $\vec{n}$ for the plane $\mathscr{P}_{frontface}$ which contains the front face of the parallelepiped shown in the figure, including the line L_1.

c) Write the simplified equation for this plane. Do the points on the line L_1 satisfy this equation?

d) Find the scalar projection h of the main diagonal of the parallelopiped along $\vec{n}$. [$|h|$ is just the distance of the front face plane from the origin, or its height if we instead think of that face as the top of the parallelopiped.]

e) Evaluate the area A of the front face of the parallelopiped, a parallelogram formed by the edges parallel to $\vec{r}_2, \vec{r}_3$.

f) Does the volume $V=|h|\,A$ of the parallelopiped equal the triple scalar product $|\vec{r}_3 \cdot (\vec{r}_1 \times \vec{r}_2)|$ as it should?

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a) simplest pt: $\vec{r}_0 = \vec{r}_1 + \vec{r}_2 = \langle 3,1,-1\rangle + \langle -1,2,1\rangle = \langle 2,3,0\rangle$

orientation: $\vec{a} = \vec{r}_3 = \langle 1,1,2\rangle$

$L_1: \vec{r} = \vec{r}_0 + t\vec{a} = \langle 2,3,0\rangle + t\langle 1,1,2\rangle = \boxed{\langle 2+t, 3+t, 2t\rangle = \langle x,y,z\rangle}$

$0 = z = 2t \to t = 0 \to x = 2+t = 2,\ y = 3+t = 3$ pt: $\boxed{(2,3,0)}$

b) $\mathscr{P}_f$ is parallel to $\vec{r}_2$ and $\vec{r}_3$ so $\vec{r}_2 \times \vec{r}_3 = \langle -1,2,1\rangle \times \langle 1,1,2\rangle \overset{\text{Maple}}{=} \langle 3,3,-3\rangle = 3\boxed{\underbrace{\langle 1,1,-1\rangle}_{\vec{n}}}$

c) $\vec{r}_0 = \vec{r}_1$ (simplest?) $= \langle 3,1,-1\rangle$ $\quad 0 = \vec{n}\cdot(\vec{r}-\vec{r}_0) = \langle 1,1,-1\rangle \cdot \langle x-3, y-1, z+1\rangle$

$= (x-3) + (y-1) - (z+1) = x+y-z-5$ $\quad \boxed{x+y-z=5}$

$(2+t)+(3+t)\ (2t) = 5+0 = 5$ ✓ line L_1 satisfies plane eqn.

d) $\vec{r}_1+\vec{r}_2+\vec{r}_3 = \langle 2,3,0\rangle + \langle 1,1,2\rangle = \langle 3,\ ,2\rangle$ main diag

$|\vec{n}| = \sqrt{1+1+1} = \sqrt{3}$, $\hat{n} = \frac{1}{\sqrt{3}}\langle 1,1,-1\rangle$, $h = \hat{n}\cdot(\vec{r}_1+\vec{r}_2+\vec{r}_3) = \frac{1}{\sqrt{3}}\langle 1,1,-1\rangle \cdot \langle 3,4,2\rangle$

$= \frac{1}{\sqrt{3}}(3+4-2) = \boxed{5/\sqrt{3}}$

e) $A = |\vec{r}_2 \times \vec{r}_3| = |3\langle 1,1,-1\rangle| = 3\sqrt{3}$, $V = A\,|h| = (3\sqrt{3})\left(\frac{5}{\sqrt{3}}\right) = \boxed{15}$] agree!

f) $|\vec{r}_3 \cdot (\vec{r}_1 \times \vec{r}_2)| = |\vec{r}_1 \cdot (\vec{r}_2 \times \vec{r}_3)| = \langle 3,1,-1\rangle \cdot \langle 3,3,-3\rangle = 9+3+3 = \boxed{15}$