MAT2500-05 22F Quiz 1 Print Name (Last, First) _____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, IDENTIFYING expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not subsitute for them.

Given the two spheres described by the equations:

$S1: x^2+y^2+z^2=4,\ S2: x^2+y^2+z^2-4x-4y+2z+8=0$

a) Complete the squares to find the coordinates of their centers and their radii: $(C1, r1)$, $(C2, r2)$.

b) Evaluate the separation distance s between the centers.

c) Comparing the sum of their radii with the separation distance, do the two spheres overlap or not? Explain.

d) Make a rough hand diagram of the cross-section plane through the symmetry axis connecting the centers of these spheres by plotting a circle of radius $r1$ at the origin of the x-y plane and a circle of radius $r2$ at a distance s along the positive x-axis. Include unit tickmarks on each axis. Be sure to label the points on the x-axis where the two circles intersect that axis with their numerical values to 2 decimal places when not integers. Does this diagram confirm your conclusion to part c)?

e) **Optional**:

Use the CSM (context sensitive menu) PlotBuilder to 3D implicitplot the second sphere, choosing the 3 coordinate ranges from -4 to 4 to get a plot window closer to the sphere. Then drop the equation for the first plot onto this plot and rotate to see if the image confirms your conclusion from part c). Explain.

▶ solution

a) $x^2+y^2+z^2=4 \rightarrow (x-0)^2+(y-0)^2+(z-0)^2=2^2$

$\boxed{C_1(0,0,0),\ r_1=2}$

$x^2+y^2+z^2+4x-4y+2z+8=0$

$x^2+4x \quad = (x+2)^2-4$

$+y^2-4y \quad +(y-2)^2-4$

$+z^2+2z+8 \quad +(z+1)^2-1+8$

$= (x-2)^2+(y-2)^2+(z+1)^2-1=0$

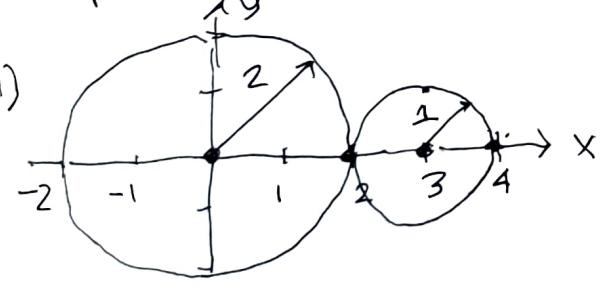
$(x-2)^2+(y-2)^2+(z+1)^2=1^2$

$\boxed{C_2(2,2,-1),\ r_2=1}$

b) $|C_1C_2|=\sqrt{(2-0)^2+(2-0)^2+(-1-0)^2}=\sqrt{4+4+1}=\sqrt{9}=\boxed{3=s}$

c) $r_1+r_2=2+1=3=s$ which can only happen if the 2 spheres intersect exactly at one point of tangency.

d)



clearly the 2 spheres must intersect in exactly one point along their comman axis of symmetry