MAT2500-05 22F Test 4 Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, IDENTIFYING expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation IF appropriate). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them, unless specifically instructed.

pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet on top of your answer sheets as a cover page, with the first test page facing up:
"During this examination, all work has been my own. I have not accessed any of the class web pages or any other sites during the exam. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date:

1. Given $\int_C \vec{F} \cdot d\vec{r} = \int_C -x^2 y^2 \, dx + x^2 y \, dy$ over the counterclockwise oriented closed curve C enclosing the region R between the curves $y = x^2$ and $y = 1$.
a) Make a diagram of this piecewise defined curve boundary and the region it encloses, labeling the points of intersection.
b) Identify the vector field $\vec{F} = \langle F_1, F_2 \rangle$ and evaluate the Green's theorem integrand $\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}$.
c) Make a new diagram illustrating the double integral over R shaded by equally spaced vertical cross-sections representing the inner integration, indicating a typical such cross-section with bullet endpoints labeled by the starting and stopping equations for the inner integration variable and evaluate (step by step by hand) the Green's theorem double integral equal to the line integral $\int_C \vec{F} \cdot d\vec{r}$.
d) Now set up the two line integrals for the two parts of this piecewise curve C and evaluate them separately, then add them together.
e) Did you manage to verify Green's theorem for this case?

2. Given the vector field $\vec{F} = \langle 3x^2 y z - 3y, x^3 z - 3x, x^3 y + 2z \rangle$:
a) Show that $\operatorname{curl}(\vec{F}) = \left\langle \frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}, \frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x}, \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right\rangle = 0$, which guarantees that this vector field admit a scalar potential function f.
b) Solve the equations $\operatorname{grad}(f) = \vec{F}$ for a potential function.
c) Use it to evaluate the line integral along any curve from $(0, 0, 0)$ to $(1, 1, 1)$.
d) Evaluate the line integral directly for the twisted cubic $\langle x, y, z \rangle = \langle t, t^2, t^3 \rangle$ for $t = 0..1$ which has the same two endpoints and correct orientation.

3. Consider the radial vector field $\vec{\boldsymbol{F}}(x, y) = \dfrac{k\,\langle x, y\rangle}{x^2 + y^2} = \dfrac{k\,\hat{\boldsymbol{r}}}{|\vec{\boldsymbol{r}}|}$ (and $\hat{\boldsymbol{r}}$ is the unit outward position vector field, while $|\vec{\boldsymbol{r}}| = \sqrt{x^2 + y^2}$), representing an inverse distance central force field.

a) Evaluate and simplify: $\operatorname{div}(\vec{\boldsymbol{F}}) = \dfrac{\partial F_1}{\partial x} + \dfrac{\partial F_2}{\partial y}$, $\operatorname{curl}(\vec{\boldsymbol{F}})_3 = \dfrac{\partial F_2}{\partial x} - \dfrac{\partial F_1}{\partial y}$.

b) Show that $f(x, y) = \dfrac{k}{2}\ \ln(x^2 + y^2)$ is a potential function for this vector field.

Do not write on this sheet. Put all work and results on the lined paper.

▶ **solution**