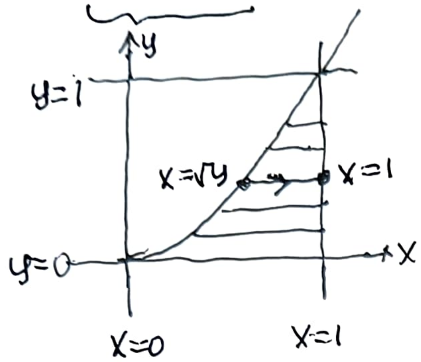
MAT2500-05 22F test 3 Answers (1)

(1) a) $\underbrace{\int_{y=0}^{y=1}\int_{x=\sqrt{y}}^{x=1}}\ \frac{ye^{x^2}}{x^3}\,dx\,dy$

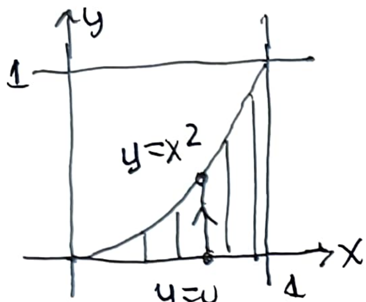
$x=\sqrt{y}..1$ while $y=0..1$



$\rightarrow\ x=\sqrt{y} \rightarrow x^2=y \rightarrow y=x^2$

$y=0..x^2$ while $x=0..1$

b)



c) $\boxed{\int_0^1 \underbrace{\int_0^{x^2} \frac{ye^{x^2}}{x^3}\,dy}\,dx} = \frac{1}{2}\int_0^1 x e^{x^2}\,dx$ d)

$u=x^2 \quad du=2x\,dx$

$x\,dx=\frac{1}{2}du$

$\begin{bmatrix} x=0 : u=0 \\ x=1 : u=1 \end{bmatrix}$

If change limits, justify

$\frac{y^2}{2x^3}e^{x^2}\Big|_{y=0}^{y=x^2} = \frac{(x^2)^2}{2x^3}e^{x^2} - 0 = \frac{x}{2}e^{x^2}$

$= \frac{1}{2}\int_{x=0}^{x=1} e^u \frac{du}{2} = \frac{1}{4}e^u\Big|_{x=0}^{x=1} = \frac{1}{4}(e^1-e^0) = \boxed{\frac{1}{4}(e-1)}$

Maple agrees!

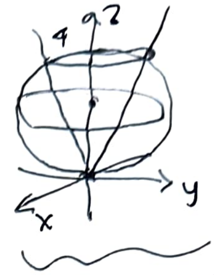
MAT2500-05 22F Test 3 Answers (2)

② a) $x^2+y^2+z^2=4z \qquad z=3\sqrt{x^2+y^2}$

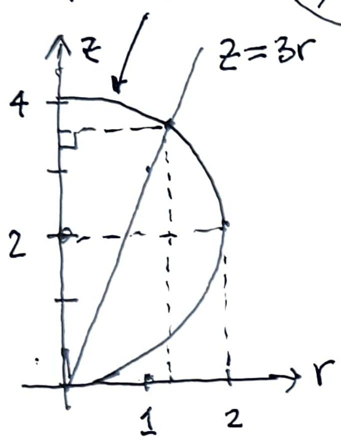
$x=0=y \nearrow$ $\hookrightarrow z^2-4z=0$

$z(z-4)=0$

$z=0,4$ intercepts, poles of sphere



b) $r^2+z^2=4z, \quad z=3r$



$r^2+(3r)^2=4(3r)$

$r^2+9r^2=12r$

$10r^2=12r$

$r(5r-6)=0$

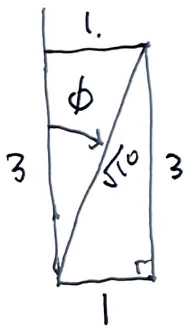
$r=0, \; r=\frac{6}{5}=1.2$

$z=3\left(\frac{6}{5}\right)=\frac{18}{5}=3.6$

$\boxed{(r,z)=\left(\frac{6}{5},\frac{18}{5}\right)}$

$\theta=0..2\pi$ revolved around z-axis but $\theta=0..\pi$ for $y\geq 0$

c)

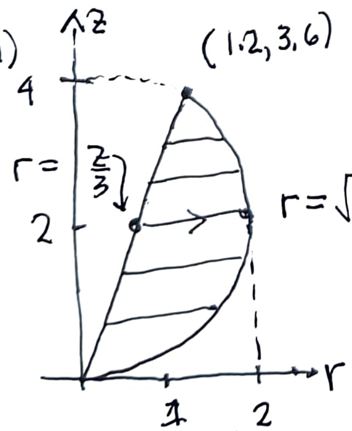


$\tan\phi=\frac{1}{3} \to \boxed{\phi=\arctan 1/3}\ \boxed{\approx 18.4^\circ}$

$\cos\phi=\frac{3}{\sqrt{10}}$

$\sin\phi=\frac{1}{\sqrt{10}}$

d)



$r=\frac{z}{3}..\sqrt{4z-z^2}$ while $z=0..\frac{18}{5}$

$r^2+z^2=4z$

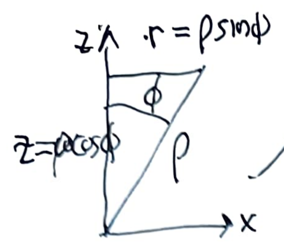
$r^2=4z-z^2$

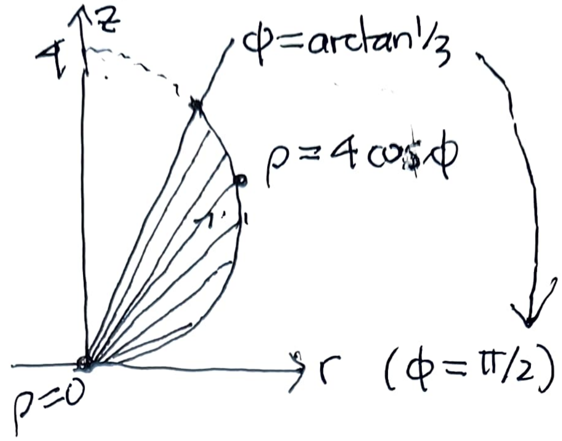
$r=\sqrt{4z-z^2}$

e) $V=\int_0^{2\pi}\int_0^{18/5}\int_{z/3}^{\sqrt{4z-z^2}} r\,dr\,dz\,d\theta = \boxed{\frac{108\pi}{25}}$ Maple

MAT2500-05 22F Test 3 Answers (3)

(2) f) $\underbrace{r^2+z^2}_{\rho^2} = \underbrace{4z}_{\rho\cos\phi} \to \rho^2 = 4\rho\cos\phi \to \boxed{\rho = 4\cos\phi}$





$\rho = 0 \,..\, 4\cos\phi$
while
$\phi = \arctan 1/3 \,..\, \pi/2$

h) $V = \int_0^{\pi}\int_{\arctan 1/3}^{\pi/2}\underbrace{\int_0^{4\cos\phi} \rho^2\sin\phi \, d\rho}\, d\phi \, d\theta$

$\frac{\rho^3}{3}\sin\phi \Big|_{\rho=0}^{\rho=4\cos\phi} = \frac{4^3\cos^3\phi}{3}\sin\phi$

$= \int_0^{\pi}\left(\int_{\arctan 1/3}^{\pi/2} \frac{64}{3}\cos^3\phi \underbrace{\sin\phi \, d\phi}_{-du}\right) d\theta$

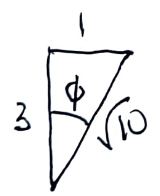
$u=\cos\phi$
$du = -\sin\phi \, d\phi$

$\int -\frac{64}{3}u^3 du = -\frac{16}{3}u^4 + C$

$= \int_0^{\pi} \underbrace{-\frac{16}{3}\cos^4\phi \Big|_{\phi=\arctan 1/3}^{\phi=\pi/2}} d\theta = \frac{4\cdot 27}{25}\int_0^{\pi} d\theta = \boxed{\frac{108\pi}{25}}$

Maple agrees!

$\frac{16}{3}\cos^4(\arctan 1/3) = \frac{16}{3}\left(\frac{3}{\sqrt{10}}\right)^4 = \frac{16\cdot 27}{100}$

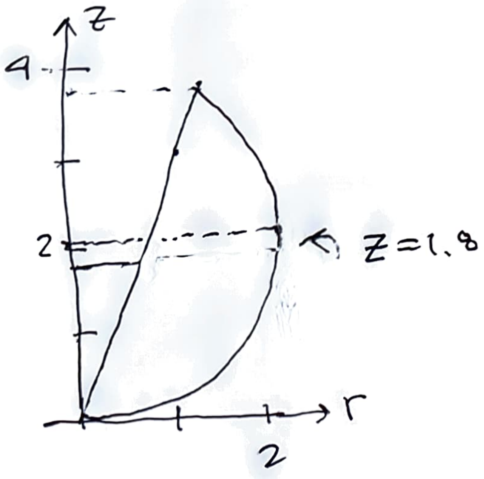


$\cos\phi = \frac{3}{\sqrt{10}}$ always simplify!

j) $M_{xy} = \int_0^{\pi}\int_{\arctan 1/3}^{\pi/2}\int_0^{4\cos\phi} (\rho\cos\phi)\rho^2\sin\phi \, d\rho \, d\phi \, d\theta \underset{\text{Maple}}{=} \frac{972\pi}{125}$

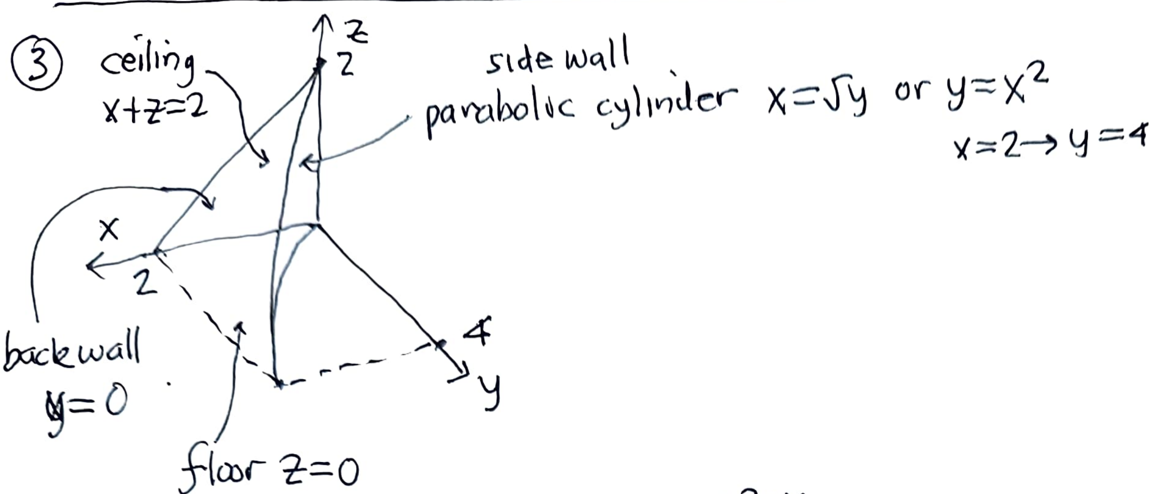
$z = \frac{M_{xy}}{V} = \frac{972\pi/125}{108\pi/25} = \boxed{\frac{9}{5} = 1.8}$

(2) j) continued

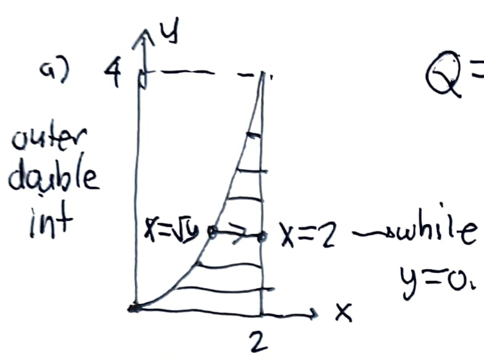


MAT2500-05 22F Test 3 Answers (5)

③



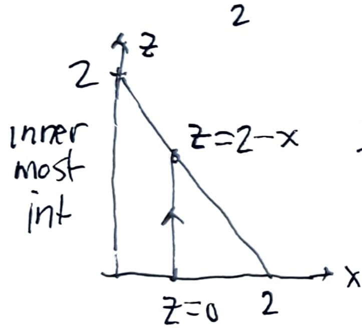
a)



$$Q=\int_0^4\underbrace{\int_{\sqrt y}^2}\underbrace{\int_0^{2-x} x+y\,dz}\,dx\,dy$$

$$= (x+y)z\Big|_{z=0}^{z=2-x} = (x+y)(2-x)$$

$$= 2x+2y-x^2-xy = (2-y)x-x^2+2y$$



$$Q=\int_0^4\int_{\sqrt y}^2 2y+(2-y)x-x^2\,dx\,dy$$

$$=\int_0^4 2yx+(2-y)\frac{x^2}{2}-\frac{x^3}{3}\Big|_{x=y^{1/2}}^{x=2}dy$$

$$=\int_0^4 2y(2-y^{1/2})+\frac{(2-y)}{2}(4-y)-\frac{1}{3}(8-y^{3/2})\,dy$$

$$=\int_0^4 4y-2y^{3/2}+\frac{8-6y+y^2}{2}+\frac{1}{3}(y^{3/2}-8)\,dy$$

$$=4\frac{y^2}{2}-2\frac{y^{5/2}}{5/2}+\frac{1}{2}\left(8y-6\frac{y^2}{2}+\frac{y^3}{3}\right)+\frac{1}{3}\left(\frac{y^{5/2}}{5/2}-8y\right)\Big|_0^4$$

$$=2y^2-\frac{4}{5}y^{5/2}+4y-\frac{3}{2}y^2+\frac{1}{6}y^3+\frac{2}{15}y^{5/2}-\frac{8}{3}y\Big|_0^4$$

$$=2\cdot16-\frac{4}{5}(32)+4\cdot4-\frac{3}{2}\cdot\overset{8}{\cancel{16}}+\frac{1}{6}(64)+\frac{2}{15}32-\frac{8}{3}\cdot4$$

$$=32\left(1-\frac{4}{5}\right)+16-24+\cancel{\frac{32}{3}}+\frac{64}{15}-\cancel{\frac{32}{3}}$$

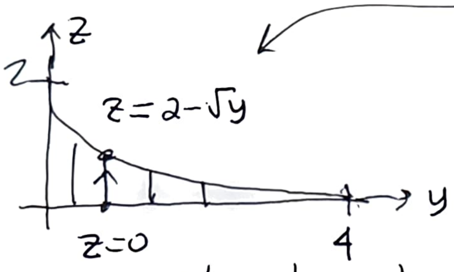
$$=32\left(\underbrace{\frac{1}{5}+\frac{2}{15}}_{\frac{5}{15}=\frac{1}{3}}\right)-8 = \frac{32-24}{3}=\frac{8}{3}$$!! yes, I did it! (Maple agrees)

MAT2500-05 22F Test 3 Answers (6)

3) b) $\left.\begin{array}{l} x+z=2 \\ x=\sqrt{y} \end{array}\right\}$ eliminate $x \longrightarrow \sqrt{y}+z=2$ $\boxed{z=2-\sqrt{y}}$

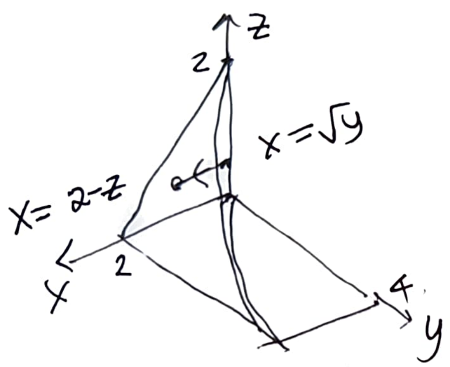
$(z-2)=\sqrt{y}$

or $y=(z-2)^2$

c) 

$z=0..\ 2-\sqrt{y}$ while $y=0..4$

outer double integral



innermost integral

$$\boxed{Q=\int_0^4\int_0^{2-\sqrt{y}}\int_{\sqrt{y}}^{2-z} x+y \; dx\,dz\,dy}$$

$\underset{\text{Maple}}{=}$ $\boxed{\frac{8}{3}}$ agrees!