

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, IDENTIFYING expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation IF appropriate). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them, except for the cross product.

## pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet on top of your answer sheets as a cover page, with the first test page facing up:

"During this examination, all work has been my own. I have not accessed any of the class web pages or any other sites during the exam. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: \_\_\_\_\_

Date: \_\_\_\_\_

1. Evaluate this double integral  $\int_0^1 \int_{\sqrt{y}}^1 \frac{y e^{x^2}}{x^3} dx dy$  by changing the order of integration following the steps:

- Make a completely labeled diagram of the 2d region of integration shaded by equally spaced linear cross-sections indicating the inner integration with a single typical such cross-section labeled by the starting and stopping equations for its variable of integration, with an arrow midway indicating the increasing direction.
- Now redraw this diagram for the opposite order of integration.
- Write down the new double integral based on your diagram.
- Evaluate it step by step by hand. Does your result agree with Maple's direct evaluation of the original double integral?

2. A solid lies inside the sphere  $x^2 + y^2 + z^2 = 4z$  and outside the cone  $z = 3\sqrt{x^2 + y^2}$  above the half plane  $y \geq 0$  as shown in the figure on the back page.

- What are the intercepts of the sphere with the positive  $z$ -axis?
- Convert these two surface equations equations to cylindrical coordinates and draw an  $r$ - $z$  half plane diagram of the two curves they reduce to, solving the pair of equations for the values  $(r, z)$  of their point of intersection away from the origin. Give their exact and decimal values (2 decimal places).
- What angle does the half line representing the cone make with the upward vertical direction (positive  $z$ -axis) in this half plane? Give your answer exactly in radians, and approximately in degrees to the nearest tenth of a degree. [This is its spherical coordinate!]
- Make a diagram of the region in the  $r$ - $z$  half plane corresponding to this solid of revolution, shading it with equally spaced linear cross-sections in the  $r$ -direction (with a typical one labeling its endpoints, arrow midway), indicating the point of intersection and its numerical coordinates.
- Write down a single triple integral in cylindrical coordinates for the volume of the solid and evaluate it exactly using Maple (no decimals).
- Convert the cylindrical coordinate equation for the sphere into spherical coordinates, solving for the radial variable.
- Make a new diagram describing the spherical coordinate integration over this plane cross-section, shaded by equally spaced radial ( $\rho$ ) cross-sections, including one labeled typical radial cross-section.
- Write down a triple integral in spherical coordinates for the volume of the solid and evaluate it step by step

by hand, using a  $u$ -substitution if necessary.

i) Check using Maple that this answer agrees with part e).

j) Now use Maple to evaluate the spherical integral of  $z$  over this solid and then divide by the volume to get the height  $Z$  of the centroid of this solid which must lie on the plane  $x=0$  by symmetry.

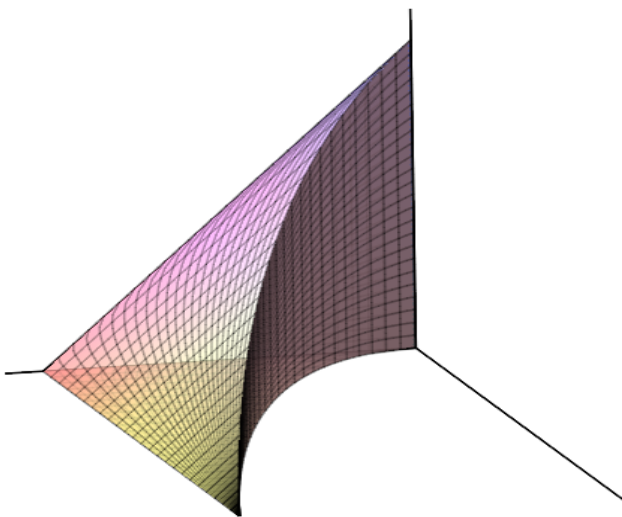
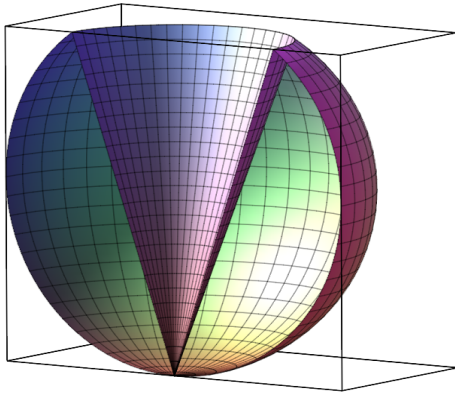
Draw a horizontal line  $z=Z$  in your  $r$ - $z$  half plane diagram, and label it by its decimal value to 2 decimal places.

3. Consider the solid region between the two surfaces  $x+z=2$ ,  $x=\sqrt{y}$  in the first octant as shown in the figure.

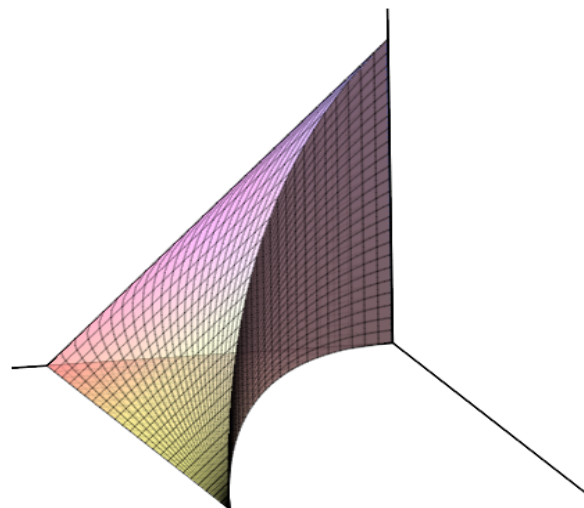
a) Set up a triple integral of the function  $f(x, y, z) = x + y$  in the order  $dz dx dy$  and evaluate it exactly by hand step by step after making a completely labeled 2d diagram representing the outer double integral, and indicate in the 3d figure a typical labeled cross section representing the innermost integral to justify the innermost limits of integration. [Make sure it is parallel to the appropriate axis.]

b) Find a direct relation between  $y$  and  $z$  for their intersection, describing the projection onto the  $y$ - $z$  plane.

c) Set up a triple integral of the function  $f(x, y, z) = x + y$  in the order  $dx dz dy$  and evaluate it exactly using Maple after making a completely labeled 2d diagram representing the outer double integral, and indicate in the 3d figure a typical labeled cross section representing the innermost integral to justify the innermost limits of integration. [Make sure it is parallel to the appropriate axis.]



innermost integral linear cross-section part a)



innermost integral linear cross-section part c)

► **solution**