MAT2500-05 22F Test 2 Print Name (Last, First) _____|_

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, IDENTIFYING expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation IF appropriate). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them, except for the cross product.

▼ pledge

1. $f(x, y, z) := x^2 + 5 y^2 + 6 y\, z + 5 z^2 - 4$, $P(2, 1, 1)$

a) Evaluate the gradient $\vec{\nabla} f(x, y, z)$ and its value at the point P, then the value of its magnitude and direction unit vector there.

b) Evaluate the directional derivative at P of this function in the direction of the origin $(0, 0, 0)$. Is it increasing or decreasing in that direction?

c) Write an equation for the level surface through this point.

d) Write a parametrized equations for the normal line to this level surface through P and find the point where it intersects the x-y plane $z = 0$.

e) Evaluate the linear approximation $L(x, y, z)$ of this function at the point P and evaluate it at the point $(2.1, 0.9, 1.1)$.

 Optional (only if you finish early, really).

f) The level surface of f through P is a tilted ellipsoid with a highest and lowest point (where the tangent plane is horizontal). Find the highest such point.

2. Consider the function $f(x, y) = x^3 - 6 x y + 8 y^3$.

a) Verify that $(0, 0)$, $\left(1, \dfrac{1}{2}\right)$ are critical points of f and evaluate f at these points.

b) Classify them as local maxima, minima or saddle points, explaining how you justified your conclusions.

3. What is the smallest surface area of a three sided open top box one can make which has fixed volume 2 (before losing its top and front side) and what dimensions give this result? Orient the box in the first octant as in the figure, so that y is the horizontal side length of the back wall and x is the side width, while z is the height, so the volume is $x\,y\,z$. State the constraint on these three dimensions and the function to be minimized. Reduce this to a problem in (x, y) and state the resulting function to be minimized and the allowed values of the two variables describing the domain over which it is to be minimized. Find the single critical point that exists in this region. Answer the word problem in an English sentence.