MAT2500-05 22F Test 1 Answers

Show all work, including mental steps means show details!

a) $\vec r = \langle t^2, \sin t - t\cos t, \cos t + t\sin t\rangle$

$\vec r\,' = \langle 2t, \underbrace{\cancel{\cos t - \cos t}}_{0} + t\sin t, \underbrace{\cancel{-\sin t + \sin t}}_{0} + t\cos t\rangle$

show cancellation!

$$\boxed{\begin{aligned} &= \langle 2t, t\sin t, t\cos t\rangle \\ &= t\langle 2, \sin t, \cos t\rangle \end{aligned}} = \vec v$$

$$v = |\vec r\,'| = |t|\sqrt{4 + \underbrace{\sin^2 t + \cos^2 t}_{1}} = \sqrt5\,|t| = \boxed{\sqrt5\, t} \text{ on } \left[0, \tfrac{3\pi}{2}\right]$$

$$\vec a = \vec r\,'' = \boxed{\langle 2, \sin t + t\cos t, \cos t - t\sin t\rangle}$$

$$\vec r\left(\tfrac{\pi}{2}\right) = \left\langle \left(\tfrac{\pi}{2}\right)^2, \underbrace{\sin\tfrac{\pi}{2}}_{1} - \tfrac{\pi}{2}\underbrace{\cos\tfrac{\pi}{2}}_{0}, \underbrace{\cos\tfrac{\pi}{2}}_{0} + \tfrac{\pi}{2}\underbrace{\sin\tfrac{\pi}{2}}_{1}\right\rangle$$

$$= \boxed{\left\langle \tfrac{\pi^2}{4}, 1, \tfrac{\pi}{2}\right\rangle}$$

$$\vec r\,'\left(\tfrac{\pi}{2}\right) = \tfrac{\pi}{2}\left\langle 2, \underbrace{\sin\tfrac{\pi}{2}}_{1}, \underbrace{\cos\tfrac{\pi}{2}}_{0}\right\rangle = \boxed{\tfrac{\pi}{2}\langle 2, 1, 0\rangle}$$

$$\left|\vec r\,'\left(\tfrac{\pi}{2}\right)\right| = \tfrac{\pi}{2}\sqrt{2^2 + 1^2} = \boxed{\sqrt5\,\tfrac{\pi}{2}}$$

$$\vec r\,''\left(\tfrac{\pi}{2}\right) = \left\langle 2, \underbrace{\sin\tfrac{\pi}{2}}_{1} + \tfrac{\pi}{2}\underbrace{\cos\tfrac{\pi}{2}}_{0}, \underbrace{\cos\tfrac{\pi}{2}}_{0} - \tfrac{\pi}{2}\underbrace{\sin\tfrac{\pi}{2}}_{1}\right\rangle$$

$$= \boxed{\left\langle 2, 1, -\tfrac{\pi}{2}\right\rangle}$$

b) $$s = \int_0^t |\vec r\,'(u)|\,du = \int_0^t \sqrt5\, u\,du = \sqrt5\,\frac{u^2}{2}\Big|_0^t = \boxed{\frac{t^2}{2}\sqrt5}$$

$$L = s\left(\tfrac{\pi}{2}\right) = \frac{\sqrt5}{2}\left(\frac{\pi}{2}\right)^2 = \boxed{\frac{\pi^2\sqrt2}{8} \approx 2.7586}$$

(≈ 2.75863)

c) $$\hat T = \frac{\vec r\,'}{|\vec r\,'|} = \frac{t\langle 2, \sin t, \cos t\rangle}{\sqrt5\, t} = \boxed{\frac{1}{\sqrt5}\langle 2, \sin t, \cos t\rangle}$$

$$\hat T\left(\tfrac{\pi}{2}\right) = \frac{1}{\sqrt5}\left\langle 2, \underbrace{\sin\tfrac{\pi}{2}}_{1}, \underbrace{\cos\tfrac{\pi}{2}}_{0}\right\rangle = \boxed{\frac{1}{\sqrt5}\langle 2, 1, 0\rangle}$$

d) $r_0 = \vec r\left(\tfrac{\pi}{2}\right) = \left\langle \tfrac{\pi^2}{4}, 1, \tfrac{\pi}{2}\right\rangle$

$a = \vec r\,'\left(\tfrac{\pi}{2}\right) = \left\langle \pi, \tfrac{\pi}{2}, 0\right\rangle$

$$\vec r = \vec r_0 + t\vec a = \left\langle \tfrac{\pi^2}{4}, 1, \tfrac{\pi}{2}\right\rangle + t\left\langle \pi, \tfrac{\pi}{2}, 0\right\rangle$$

$$= \boxed{\left\langle \tfrac{\pi^2}{4} + \pi t, 1 + \tfrac{\pi}{2}t, \tfrac{\pi}{2}\right\rangle = \langle x, y, z\rangle}$$

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e) $\vec{b}(\frac{\pi}{2}) = \vec{r}'(\frac{\pi}{2}) \times \vec{r}''(\frac{\pi}{2}) = \frac{\pi}{2} \langle 2,1,0\rangle \times \langle 2,1,-\frac{\pi}{2}\rangle$

$\overset{\text{Maple}}{=} \frac{\pi}{2}\langle -\frac{\pi}{2}, \pi, 0\rangle = \langle -\frac{\pi^2}{4}, \frac{\pi^2}{2}, 0\rangle = \frac{\pi^2}{4}\langle -1,2,0\rangle$

$|\vec{b}(\frac{\pi}{2})| = \frac{\pi^2}{4}\sqrt{1^2+2^2} = \frac{\sqrt{5}}{4}\pi^2$

$\vec{B}(\frac{\pi}{2}) = \frac{\vec{b}(\pi/2)}{|\vec{b}(\pi/2)|} = \frac{\frac{\pi^2}{4}\langle -1,2,0\rangle}{\sqrt{5}\frac{\pi^2}{4}} = \boxed{\frac{1}{\sqrt{5}}\langle -1,2,0\rangle}$

f) $\hat{N}(\frac{\pi}{2}) = \hat{B}(\frac{\pi}{2}) \times \hat{T}(\frac{\pi}{2}) = \frac{1}{\sqrt{5}}\langle -1,2,0\rangle \times \frac{1}{\sqrt{5}}\langle 2,1,0\rangle$

$\overset{\text{Maple}}{=} \boxed{\langle 0,0,-1\rangle}$

Note $\langle 0,0,a\rangle$ is a unit vector only if $a=\pm 1$!

g) $\kappa(\frac{\pi}{2}) = \frac{|\vec{b}(\pi/2)|}{|\vec{r}'(\frac{\pi}{2})|^3} = \frac{\frac{\sqrt{5}}{4}\pi^2}{(\sqrt{5}\frac{\pi}{2})^3} = \frac{\sqrt{5}\pi^2\cdot 8}{4\cdot 5\sqrt{5}\pi^3} = \boxed{\frac{2}{5\pi}}$

numerator clearly has length $\sqrt{4+1}=\sqrt{5}$ so also obviously a unit vector

h) $a_{\hat{T}}(\frac{\pi}{2}) = \hat{T}(\frac{\pi}{2})\cdot\vec{r}''(\frac{\pi}{2}) = \frac{1}{\sqrt{5}}\langle 2,1,0\rangle\cdot\langle 2,1,-\frac{\pi}{2}\rangle$

$= \frac{1}{\sqrt{5}}(4+1+0) = \frac{5}{\sqrt{5}} = \boxed{\sqrt{5}}$

$a_{\hat{N}}(\frac{\pi}{2}) = \hat{N}(\frac{\pi}{2})\cdot\vec{r}''(\frac{\pi}{2}) = \langle 0,0,-1\rangle\cdot\langle 2,1,-\frac{\pi}{2}\rangle = 0+0+\frac{\pi}{2}$

$= \boxed{\frac{\pi}{2}}$

not requested:

check: $|\vec{r}''(\frac{\pi}{2})| = |\langle 2,1,-\frac{\pi}{2}\rangle| = \sqrt{2^2+1^2+(\frac{\pi}{2})^2} = \boxed{\sqrt{5+\frac{\pi^2}{4}}}$

$\sqrt{a_{\hat{N}}(\frac{\pi}{2})^2 + a_{\hat{T}}(\frac{\pi}{2})^2} = \sqrt{(\frac{\pi}{2})^2+(\sqrt{5})^2} = \sqrt{5+\frac{\pi^2}{4}}$