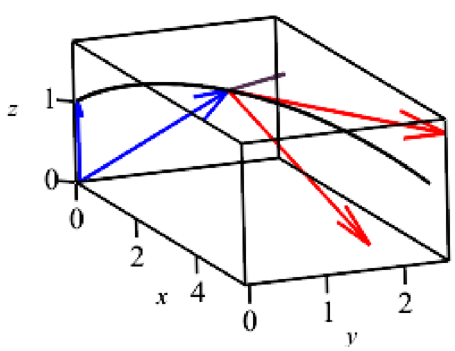
MAT2500-05 22F Test 1 Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, IDENTIFYING expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation IF appropriate). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them, except for the cross product.

pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet on top of your answer sheets as a cover page, with the first test page facing up:
"During this examination, all work has been my own. I have not accessed any of the class web pages or any other sites during the exam. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date:



The parametrized curve segment

$\vec{r}(t) = \langle t^2, \sin(t) - t\cos(t), \cos(t) + t\sin(t)\rangle \quad , 0 \le t \le \frac{3\pi}{4}$.

is shown in the figure together with $\vec{r}(0)$ and $\vec{r}\left(\frac{\pi}{2}\right)$ and the first and second derivatives at the latter point on the curve.

a) Evaluate and simplify $\vec{v}(t) = \vec{r}\,'(t)$, $v(t) = |\vec{r}\,'(t)|$, $\vec{a}(t) = \vec{r}\,''(t)$ and their values at $t = \frac{\pi}{2}$ including also $\vec{r}\left(\frac{\pi}{2}\right)$.

b) Your expression for the speed $v(t)$ can be simplfied to easily find its antiderivative. This allows you to find exactly evaluate the arclength function starting at $t = 0$. Use it to evaluate the exact arclength of the curve over the interval $0 \le t \le \frac{\pi}{2}$ and its numerical approximation to 4 decimal places.

c) Evaluate the unit tangent $\hat{T}(t)$ and $\hat{T}\left(\frac{\pi}{2}\right)$.

d) Write the parametrized equations of the tangent line through $\vec{r}\left(\frac{\pi}{2}\right)$.

e) Evaluate $\vec{b}\left(\frac{\pi}{2}\right) = \vec{r}\,'\left(\frac{\pi}{2}\right) \times \vec{r}\,''\left(\frac{\pi}{2}\right)$. Then evaluate and simplify the unit binormal $\hat{B}\left(\frac{\pi}{2}\right) = \hat{b}\left(\frac{\pi}{2}\right)$ obtained by normalizing it.

f) Evaluate and simplify the unit normal $\hat{N}\left(\frac{\pi}{2}\right) = \hat{B}\left(\frac{\pi}{2}\right) \times \hat{T}\left(\frac{\pi}{2}\right)$.

g) Evaluate the curvature

$$\kappa\left(\frac{\pi}{2}\right) = \frac{\left|\vec{r}\,'\left(\frac{\pi}{2}\right) \times \vec{r}\,''\left(\frac{\pi}{2}\right)\right|}{\left|\vec{r}\,'\left(\frac{\pi}{2}\right)\right|^3} .$$

h) Evaluate the scalar tangential projection $a_{\hat{T}}\left(\frac{\pi}{2}\right)$ along $\hat{T}\left(\frac{\pi}{2}\right)$ of the acceleration $\vec{a}\left(\frac{\pi}{2}\right) = \vec{r}\,''\left(\frac{\pi}{2}\right)$ and its scalar normal projection $a_{\hat{N}}\left(\frac{\pi}{2}\right) = \hat{N}\left(\frac{\pi}{2}\right) \cdot \vec{a}\left(\frac{\pi}{2}\right)$ exactly.

For spacecurve (not needed) load plots, for vector function derivatives load:

```
> with(plots) :  with(Student[VectorCalculus]) : BasisFormat(false) :
> r(t) := ⟨cos(t), sin(t), t^2⟩; r'(t); r''(t)
```

▶ solution