MAT1505-05 22F Quiz 8 Print Name (Last, First) ________________ , ________________

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, IDENTIFYING expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC). INDICATE where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them.

1. Is $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sqrt{n}}{2+3n}$ absolutely convergent, conditionally convergent or divergent? Justify your claim.

2. Use the ratio test to determine whether the series is convergent or divergent: $\sum_{n=0}^{\infty}\frac{(-3)^n}{(2n+1)!}$

3. a) Verify that the series $S=\sum_{n=1}^{\infty}\left(\frac{n^{10}}{(-10)^{n+1}}\right)$ converges by the ratio test.

b) Using the alternating series (next term) estimate for the maximum absolute value of the remainder, if one truncates this series after the nth term (let S_n be the nth partial sum), what is the smallest value of n for which this approximates the series accurately to within 0.00005 (namely $0.5\cdot 10^{-4}$) ?
[Hint: n is less than 20.]

c) Use Maple to compare your estimate to the actual error $|S-S_n|$ for this value of n. Confirm that this is less than your estimate. [State values of S and S_n and their difference to at least 6 decimal places.]

▶ **solution**

1 $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n^{1/2}}{2+3n}$

alternating series

for large n: $\frac{n^{1/2}}{3n}=\frac{1}{3n^{1/2}} \rightsquigarrow p=\frac{1}{2}$ series , $|a_n|$ decreases to 0

so converges by alternating series test

but abs. value series is divergent p-series since $p<1$

so $\boxed{\text{converges conditionally}}$

2 $\sum_{n=0}^{\infty}\frac{(-3)^n}{(2n+1)!}=\sum_{n=0}^{\infty}\frac{(-1)^n 3^n}{(2n+1)!}$ [factorials beat exponentials (G.S.'s)]

$\left|\frac{a_{n+1}}{a_n}\right|=\frac{\frac{3^{n+1}}{(2(n+1)+1)!}}{\frac{3^n}{(2n+1)!}}=\frac{3^{n+1}}{3^n}\cdot\frac{(2n+1)!}{(2n+3)!}=3^1\cdot\frac{\cancel{(2n+1)!}}{(2n+3)(2n+2)\cancel{(2n+1)!}}$

$=\frac{3}{(2n+3)(2n+2)}\xrightarrow[n\to\infty]{}0<1$

so this series $\boxed{\text{converges absolutely}}$

MAT1505-US 22F Quiz 8

③ a) $S = \sum_{n=1}^{\infty} \frac{n^{10}}{(-10)^{n+1}} = \sum_{n=1}^{\infty} (-1)^n \frac{n^{10}}{10^{n+1}}$

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{\frac{(n+1)^{10}}{10^{(n+1)+1}}}{\frac{n^{10}}{10^{n+1}}} = \frac{(n+1)^{10}}{n^{10}} \cdot \frac{10^{n+1}}{10^{n+2}} = \left(\frac{n+1}{n}\right)^{10} \cdot \frac{1}{10}$$

$$= \Big(1 + \underbrace{\tfrac{1}{n}}_{\to 0}\Big)^{10} \cdot \frac{1}{10} \xrightarrow[n\to\infty]{} \frac{1}{10} < 1$$

looks more and more like a G.S.
with ratio $\frac{1}{10}$ as n increases
so converges (absolutely)

next term estimate:

b). $|a_{n+1}| = \frac{(n+1)^{10}}{10^{n+2}} < 0.5 \times 10^{-4}$

or $\frac{10^{n+2}}{(n+1)^{10}} > 2 \times 10^4$

solve equality with Maple:
$n = 14.087 \longrightarrow \boxed{15 = n}$
first integer satisfying inequality
so S_{15} is the desired approximation

Maple:

c) $S = 0.144\,7199$

$S_{15} = \sum_{n=1}^{15} (-1)^n \frac{n^{10}}{10^{n+1}} = 0.144\,7292$

$|S - S_{15}| = 9.2837 \cdot 10^{-6} < .00005$ as desired.

if inequality confusing just evaluate terms:
note $|a_{15}| = 0.000057 > 0.00005$
$|a_{16}| = 0.000011 < 0.00005$

so S_{15} is the first partial sum
for which next term is less than
desired precision