MAT1505-05 22F Quiz 6   Print Name (Last, First) _____ , _____|___

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, IDENTIFYING expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC). INDICATE where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them.

1. a) Consider the arclength function $s(x)$ for the graph $y = 1 + \ln(x) = f(x)$ zeroed at the obvious special point $x = 1$ (namely $s(1) = 0$). Write down the definite integral required to evaluate it.
b) Let Maple evaluate this in two steps: first find Maple's choice of antiderivative, then evaluate its difference at the appropriate limits.
[If you try to let Maple evaluate the definite integral directly you have to first do: $> assume(x > 0);$]
c) What is the value of this function at $x = 2$?
d) Plot f and s together for $1 \le x \le 3$ and roughly estimate the value of x where they intersect.
e) Find the numerial value of x where they intersect to 6 decimal places and evaluate the arclength function there to the same accuracy.

2. Find the area between the graph of the hyperbolic tangent $y = \tanh(x)$ and its asymptote $y = 1$ on the positive x-axis, setting up the necessary limit, using Maple to find the antiderivative, and then using it to state the limit you need to consider to define this area, and then use Maple to evaluate that limit. The result turns out to be really simple!
Optional.

The arctan function has a similar looking graph with asymptote $y = \dfrac{\pi}{2}$. Is the corresponding area between it and its asymptote on the positive x-axis finite or not?

Recall $> plot([f(x), s(x)], x = 1..3, gridlines = true)$

▶ **solution**

① a) $f(x) = 1 + \ln x \quad f'(x) = 0 + \frac{1}{x} = \frac{1}{x} \quad (x > 0)$

$\sqrt{1 + f'(x)^2} = \sqrt{1 + \frac{1}{x^2}} = \sqrt{\frac{x^2 + 1}{x^2}} = \frac{\sqrt{1 + x^2}}{x}$ "simplest form"
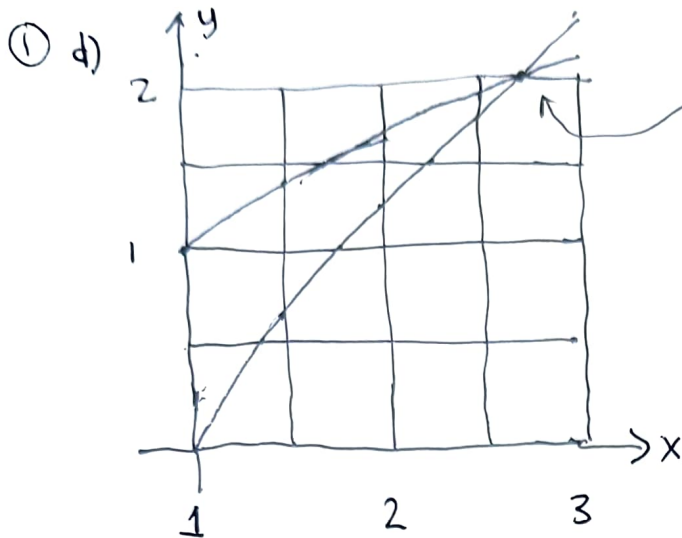
$s(x) = \int_0^x \sqrt{1 + f'(t)^2}\, dt = \boxed{\int_0^x \frac{\sqrt{1 + t^2}}{t}\, dt}$

$\dfrac{\sqrt{1 + x^2}}{x} \xrightarrow[\text{Maple}]{\text{antiderivative}} \sqrt{x^2 + 1} - \text{arctanh} \dfrac{1}{\sqrt{x^2 + 1}}$ so:

b) $s(x) = \sqrt{t^2 + 1} - \text{arctanh} \dfrac{1}{\sqrt{t^2 + 1}} \Big|_0^x = \sqrt{x^2 + 1} - \text{arctanh} \dfrac{1}{\sqrt{x^2 + 1}} - \left(\sqrt{0 + 1} - \text{arctanh} \dfrac{1}{\sqrt{2}}\right)$

$= \boxed{\sqrt{x^2 + 1} - \text{arctanh} \dfrac{1}{\sqrt{x^2 + 1}} - 1 + \text{arctanh} \tfrac{1}{\sqrt{2}}}$

c) $\boxed{s(2) = \sqrt{5} - \text{arctanh} \dfrac{1}{\sqrt{5}} - 1 + \text{arctanh} \tfrac{1}{\sqrt{2}}} \approx 1.222$
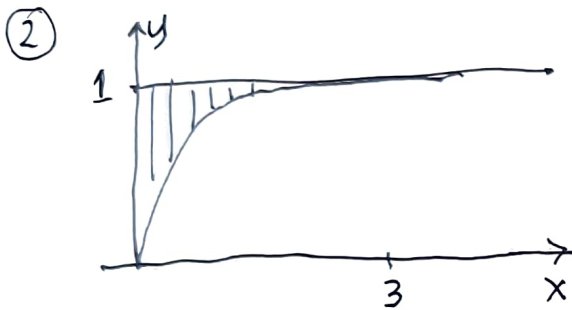
① d)

about $(2.7, 2.)$

$x \approx 2.713267$

e) $s(x) = f(x) \xrightarrow[\text{solve}]{\text{numerical}}$ $s \approx 1.998154$
      Maple

②

$$A = \boxed{\int_0^\infty 1 - \tanh x \, dx} = \boxed{\lim_{t \to \infty} \int_0^t 1 - \tanh x \, dx}$$

$$\underset{\text{Maple}}{=} \boxed{\lim_{t \to \infty} (t - \ln \cosh t)} \underset{\text{Maple}}{=} \boxed{\ln 2} \approx 0.69$$

not requested :

$$\overset{\text{"}}{\underset{t \to \infty}{\lim}} \quad t - \ln\left(\frac{e^t + e^{-t}}{2}\right)$$

$$\underbrace{= \ln\left(\frac{e^t}{2}(1 + e^{-2t})\right)} = \underbrace{\ln \frac{e^t}{t}}_{} - \ln 2 + \ln(1 + e^{-2t})$$

$$= \lim_{t \to \infty} \quad t - (t - \ln 2 + \ln(1 + e^{-2t}))$$

$$= \lim_{t \to \infty} \quad \ln 2 + \underbrace{\ln(1 + e^{-2t})}_{\to 0} \quad = \ln 2 \, !$$

not really
a difficult limit to do
BUT being able to use
a tool like Maple allows
you to get answers when
not so simple!