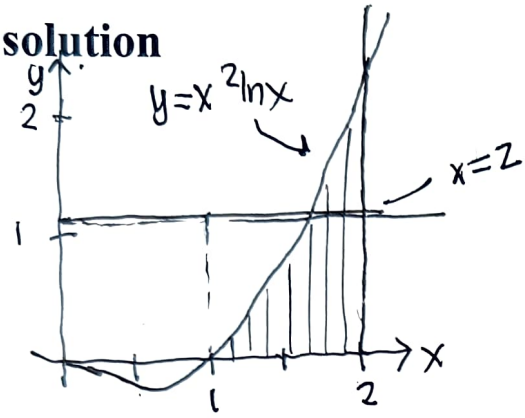
MAT1505-05 22F Quiz 4 Print Name (Last, First) _____, _____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). INDICATE where technology is used and what type (Maple, GC).

1. a) Use integration by parts to find an antiderivative of $f(x) = x^2 \ln(x)$, showing all steps.
b) Use that result to evaluate the (positive) area enclosed between the its graph and the x-axis and the line $x = 2$, first showing how this interval is obtained. Start by plotting the function in an appropriate window that shows this region and give a rough completely labeled sketch of what you see.
c) What is the average value of this $f(x)$ over this interval?
d) Include the graph of the constant function equal to the average value in your previous sketch.

▶ **solution**

b)



$x^2 \ln x = 0 \rightarrow x = 0$ or $\ln x = 0$, $x = 1$

so $A = \int_1^2 x^2 \ln x \, dx$

a) $\int \underbrace{x^2}_{dv} \underbrace{\ln x}_{u} \, dx = \frac{x^3}{3} \ln x - \int \frac{x^3}{3} \frac{1}{x} dx$

$u = \ln x \quad dv = x^2 dx$

$du = \frac{1}{x} dx \quad v = \frac{x^3}{3}$

$\frac{1}{3} \int x^2 dx = \frac{1}{9} x^3$

$= \boxed{\frac{x^3}{3}\left(\ln x - \frac{1}{3}\right) + C}$

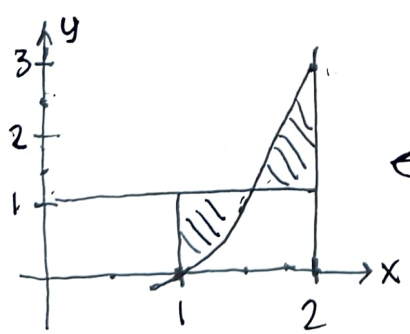
"an antiderivative"

c) $f_{avg} = \frac{1}{1} \int_1^2 x^2 \ln x \, dx = \frac{x^3}{3}\left(\ln x - \frac{1}{3}\right)\Big|_1^2$

$= \frac{8}{3}\left(\ln 2 - \frac{1}{3}\right) - \frac{1}{3}\left(\ln 1 - \frac{1}{3}\right)$ ($\ln 1 = 0$)

$= \frac{8}{3}\ln 2 - \frac{8}{9} + \frac{1}{9}$ ($-\frac{7}{9}$)

$= \boxed{\frac{8}{3}\ln 2 - \frac{7}{9}} \approx 1.071$



← areas seem equal

(previous sketch badly drawn)