MAT1505-05 22F Quiz 3 Print Name (Last, First) ____________________ , ____________________

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). INDICATE where technology is used and what type (Maple, GC).

Find the volume of a frustrum of a right circular cone with height h , lower base R, and top radius r. by following the steps.

a) Consider the vertical plane cross-section through its axis of symmetry as shown in the right figure and label the two endpoints of the line representing the slanted side face of the solid by their coordinates based on the dimensions shown in the figure to the left.

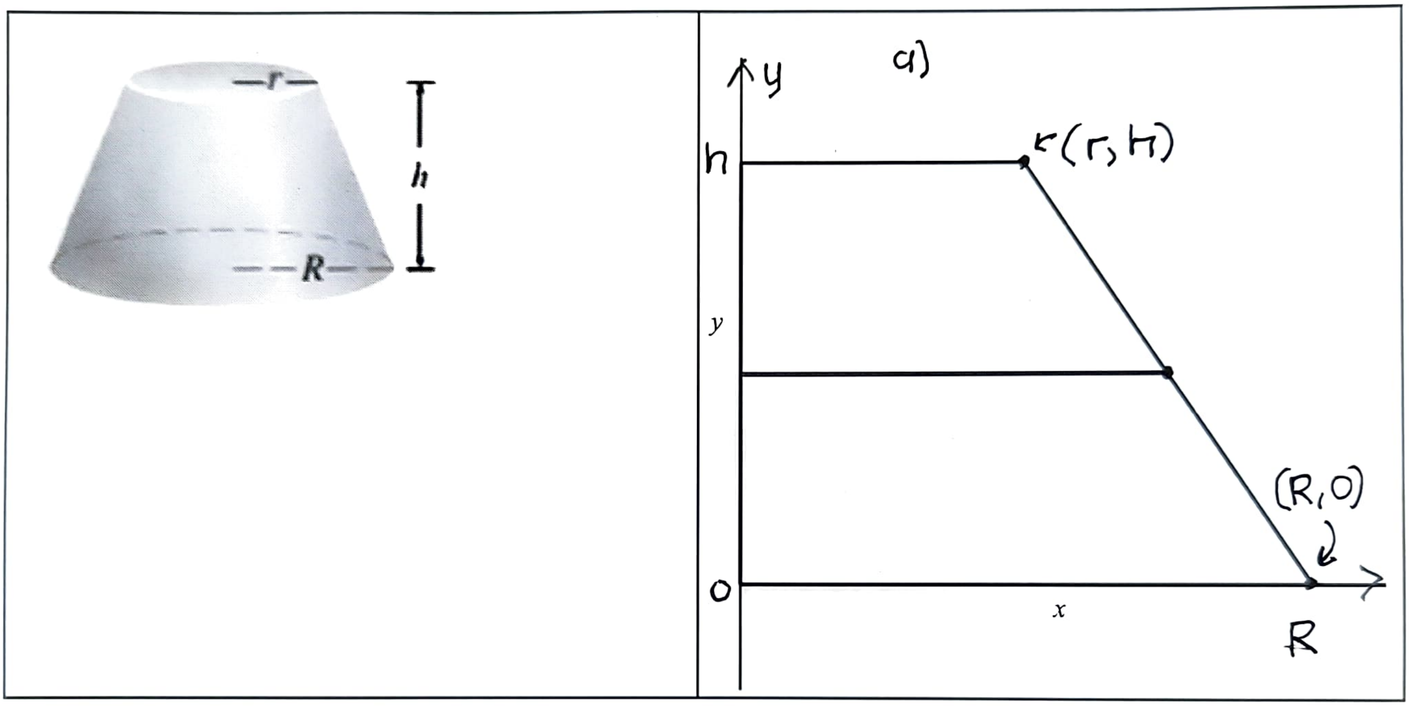
b) Write an equation for the line containing these two points and solve for x as a function of y.

c) Set up an integral formula in simplified form for the volume of revolution corresponding to revolving this figure about the y-axis

d) Evaluate this integral by hand step by step and simplify the resulting formula.

e) Does this agree with the formula you can easily find on the internet?

f) Enter the integral in Maple and evaluate it, then "Simplify" it. Does this agree with your answer?



b) slope : $m = \frac{h-0}{r-R} = -\frac{h}{R-r} < 0 \to y - 0 = m(x-R) = -\frac{h}{R-r}(x-R)$ $\quad y = +h\frac{(R-x)}{(R-r)}$

$R - x = \frac{R-r}{h}y \to x = R - (R-r)\frac{y}{h} = \boxed{\frac{hR-(R-r)y}{h} = x}$

c) $V = \int_0^h A(y)\,dy = \int_0^h \pi X(y)^2\,dy = \int_0^h \frac{(hR-(R-r)y)^2}{h^2}\,dy$

d) $= \boxed{\frac{\pi}{h^2}\int_0^h [h^2R^2 - 2hR(R-r)y + (R-r)^2y^2]\,dy}$ simplified integral,

$\boxed{\text{e): enter in Maple}}$

$= \frac{\pi}{h^2}\left(h^2R^2y - hR(R-r)y^2 + (R-r)^2 y^3/3\right)\Big|_0^h$

$= \frac{\pi}{h^2}\left(h^3R^2 - h^3R(R-r) + (R-r)^2h^3/3\right) = \pi h\left(\cancel{R^2} - \cancel{R^2} + Rr + \frac{R^2-2Rr+r^2}{3}\right)$ $\quad \frac{1}{3}Rr$

$= \boxed{\frac{\pi h}{3}(R^2 + Rr + r^2)}$ e) yes! agrees with internet and Maple!