

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). INDICATE where technology is used and what type (Maple, GC).

An internet search finds the claim: "US adult male heights are on average 70 inches (5'10") with a standard deviation of 4 inches." The fraction  $F$  of such adult males within one standard deviation of the mean value ("average") for a normal distribution is given by the following definite integral

$$F = \int_{66}^{74} \frac{1}{4\sqrt{2\pi}} e^{-\frac{1}{2} \cdot \left(\frac{x-70}{4}\right)^2} dx$$

- Use Maple to evaluate this exactly and then approximate the result to 3 decimal places.
- Use the change of variable to the "standard variable"  $u = \frac{x-70}{4}$  to transform this definite integral to its standard form in terms of that new variable (which measures the deviation from the average value in multiples of the standard deviation).
- Use Maple to evaluate the new integral exactly and then approximate it to 3 decimal places.
- They should agree. If not find your error.
- What percent of this population has a height within one standard deviation of the mean value of 70? Answer in a complete English sentence.

► **solution**

a)  $F = \int_{66}^{74} \frac{1}{4\sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{x-70}{4}\right)^2} dx \stackrel{\text{Maple}}{=} \boxed{\text{erf}\left(\frac{\sqrt{2}}{2}\right)} \approx 0.682689$   
 $\approx \boxed{0.683}$

b)  $\left( \begin{array}{l} u = \frac{x-70}{4} \\ du = \frac{dx}{4}, dx = 4du \end{array} \right) F = \int_{-1}^1 \frac{1}{4\sqrt{2\pi}} e^{-u^2} (4du) = \boxed{\frac{1}{\sqrt{2\pi}} \int_{-1}^1 e^{-u^2} du}$

$\left( \begin{array}{l} x=66 \rightarrow u = \frac{66-70}{4} = -1 \\ x=74 \rightarrow u = \frac{74-70}{4} = 1 \end{array} \right) \uparrow$

(c)  $= \boxed{\text{erf}\left(\frac{\sqrt{2}}{2}\right)}$  agree! (d)  
 $\approx \boxed{0.683}$

e) About 68.3% (roughly 2/3) of the US adult male population has a height within one standard deviation of the average value of 70 inches.