MAT1505-05 22F Quiz 1   Print Name (Last, First) ______________________________

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). INDICATE where technology is used and what type (Maple, GC).

Consider the velocity function $v(t) = 10 - 5\,t$ on the interval $0 \le t \le 3$.

a) Find the times at which the velocity is zero, positive and negative and make a labeled diagram illustrating this situation on the given interval. Over what interval is the displacement $s(t)$ increasing? Decreasing?

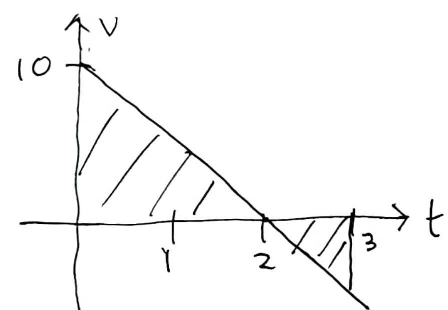
b) Evaluate the exact total displacement over this time interval, showing your work step by step and give the numerical approximation to 3 decimal places.

c) Evaluate the exact distance traveled by setting up the two separate integrals over which the velocity has a given sign using technology to evaluate them and their sum, and its numerical approximation to 3 decimal places.

d) How much distance was traveled while moving in the direction of the positive s axis? Give both the exact value and the numerical approximation to 3 decimal places.

▶ solution

a) $V(t) = 10 - 5t = 0 \rightarrow t = 10/5 = 2$ $\boxed{\text{velocity is zero at } t=2}$



$\boxed{\begin{array}{l} S(t) \text{ is increasing over } 0 \le t < 2 \text{ where } V > 0. \\ S(t) \text{ is decreasing over } 2 < t \le 3 \text{ where } V < 0 \end{array}}$

motion comes to rest at $t = 2$ after initially moving to the right on the s axis, then moves left.

$V>0$ on $0 \le t < 2$ | $V>0$ on $2 < t \le 3$

$V=0$ at $t=2$

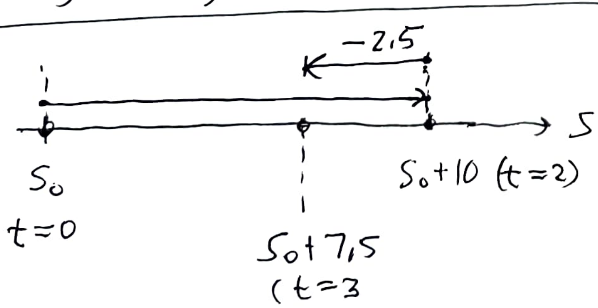
$\underbrace{\Delta S(t) = \int_0^t v(u)\,du}$

b) $\Delta S = \int_0^3 v(t)\,dt = \int_0^3 10-5t\,dt = \left(10t - \frac{5 \cdot t^2}{2}\right)\Big|_0^3 = 30 - \frac{45}{2} = \frac{60-45}{2} = \boxed{\frac{15}{2} = 7.5}$

c) $d = \int_0^3 |v(t)|\,dt = \int_0^2 10-5t\,dt - \int_2^3 10-5t\,dt = \left(10t - \frac{5t^2}{2}\right)\Big|_0^2 - \left(10t - \frac{5t^2}{2}\right)\Big|_2^3$

$= \underbrace{(20-10) - 0}_{10} \underbrace{- [\underbrace{(30 - \tfrac{45}{2}) - (20-10)}_{15/2 \,-\, 10 \,=\, -5/2}]}_{+\,5/2} = \boxed{\frac{25}{2} = 12.5}$

d) $\boxed{10 \text{ units forward}, \quad 5/2 = 2.5 \text{ units backwards}}$ for a net 7.5 units forwards.



illustrates what we have found for motion (not requested)