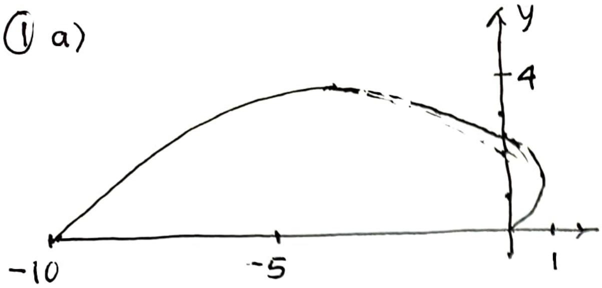
MAT 1505-05 22F Final Exam

① a)



b) $r=\theta^2,\ r' = 2\theta$

$r^2+r'^2 = (\theta^2)^2+(2\theta)^2 = \theta^4+4\theta^2$

$= \theta^2(\theta^2+4)$

$\sqrt{r^2+r'^2} = \theta(\theta^2+4)^{1/2}$ for $\theta \geq 0$

$L = \int_0^\pi \theta\sqrt{\theta^2+4}\,d\theta$

$u=\theta^2+4$

$du = 2\theta\, d\theta$

$\frac{du}{2} = \theta\, d\theta$

$\int \theta\sqrt{\theta^2+4}\,d\theta = \int u^{1/2}\frac{du}{2} = \frac{1}{2}\frac{u^{3/2}}{3/2}+C$

$= \frac{1}{3}(\theta^2+4)^{3/2}+C$

$L = \frac{1}{3}(\theta^2+4)^{3/2}\Big|_0^\pi$

$= \frac{1}{3}\left[(\pi^2+4)^{3/2} - 4^{3/2}\right]$

$= \boxed{\frac{1}{3}\left[(\pi^2+4)^{3/2}-8\right]}$

c) $A = \int_0^\pi \frac{1}{2}(\theta^2)^2 d\theta$

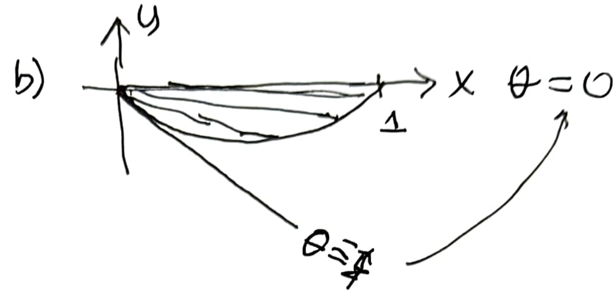
$= \int_0^\pi \frac{1}{2}\theta^4 d\theta = \frac{\theta^5}{2\cdot 5}\Big|_0^\pi = \boxed{\frac{\pi^5}{10}}$

② a) $r=\cos\theta+\sin\theta = 0$

$\sin\theta = -\cos\theta$

$\tan\theta = -1 \rightarrow \boxed{\theta = -\frac{\pi}{4}}$

b)



$\theta = -\frac{\pi}{4} .. 0$ so

$A = \int_{-\frac{\pi}{4}}^0 \frac{1}{2} r(\theta)^2 d\theta$

$= \int_{-\frac{\pi}{4}}^0 \frac{1}{2}(\cos\theta+\sin\theta)^2 d\theta$

$\frac{1}{2}(\underbrace{\cos^2\theta+\sin^2\theta}_{1}+2\cos\theta\sin\theta)$

$= \frac{1}{2}(1+2\sin\theta\cos\theta)$

$= \int_{-\frac{\pi}{4}}^0 \frac{1}{2}(1+2\sin\theta\cos\theta)\,d\theta$

$= \int_{-\frac{\pi}{4}}^0 \frac{1}{2}d\theta + \int_{-\frac{\pi}{4}}^0 \underbrace{\sin\theta}_{u}\,\underbrace{\cos\theta\,d\theta}_{du}$

$\rightarrow \frac{u^2}{2} = \frac{\sin^2\theta}{2}$

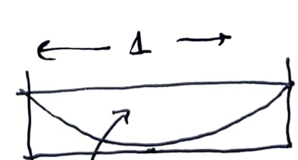
$= \frac{1}{2}(\theta+\sin^2\theta)\Big|_{-\frac{\pi}{4}}^0$

$= \frac{1}{2}\left(0-\left(-\frac{\pi}{4}\right)+0-\sin^2\frac{\pi}{4}\right)$

$= \frac{1}{2}\left(\frac{\pi}{4}-\left(\frac{1}{\sqrt{2}}\right)^2\right) = \boxed{\frac{\pi}{8}-\frac{1}{4}}$

≈ 0.1427

c)



$0.2 = 1/5 \quad A_{rect} = 0.2$

$A_{circ} \approx 0.143 \approx \frac{3}{4}(0.2) = 0.15$

seems about right, yes!

MAT1505-05 22F Final Exam (2)

② d) $\underbrace{x^2+y^2}_{r^2} = \underbrace{x}_{r\cos\theta} + \underbrace{y}_{r\sin\theta} \to r^2 = r\cos\theta + r\sin\theta = r(\cos\theta+\sin\theta)$

divide by r!

$r = \cos\theta + \sin\theta$ ✓

e) $x^2+y^2=x+y \xrightarrow{y=0} x^2=x \Rightarrow x^2-x=0 \quad x(x-1)=0$

$x=0,1$ intercepts (obvious in figure!)

f) $x^2+y^2=x+y$

$y^2-y+(x^2-x)=0 \to y = \dfrac{1\pm\sqrt{1-4(x^2-x)}}{2}$

$\downarrow$

$y = \dfrac{1-\sqrt{1-4(x^2-x)}}{2}$ (lower semicircle solution)

$A = \int_0^1 -y\,dx = \int_0^1 \dfrac{-1+\sqrt{1-4(x^2-x)}}{2}\,dx \underset{\text{Maple}}{=} -\frac{1}{4}+\frac{\pi}{8}$ ✓

③ a) $x = 2t^3+3t$ b) $x' = 6t^2+3$ $\quad \dfrac{dy}{dx} = \dfrac{y'}{x'} = \dfrac{4-10t}{6t^2+3}$

$y = 4t-5t^2 \quad y' = 4-10t$

b) $x(1) = 2+3 = 5$

$y(1) = 4-5 = -1$

$\frac{dy}{dx}(1) = \frac{4-10}{6+3} = -\frac{6}{9} = -\frac{2}{3}$

$y = y(1) + \frac{dy}{dx}(1)\,(x-x(1))$

$= -1 + \left(-\frac{2}{3}\right)(x-5)$

$= -1 + \frac{10}{3} - \frac{2}{3}x = \frac{7}{3} - \frac{2}{3}x$

$\boxed{y = \frac{7}{3}-\frac{2}{3}x} = \frac{7-2x}{3} = 0 \to x = \frac{7}{2} = \boxed{3.5}$

$\downarrow$

$x=0 \to y = \frac{7}{3} = 2\frac{1}{3} \approx \boxed{2.33}$

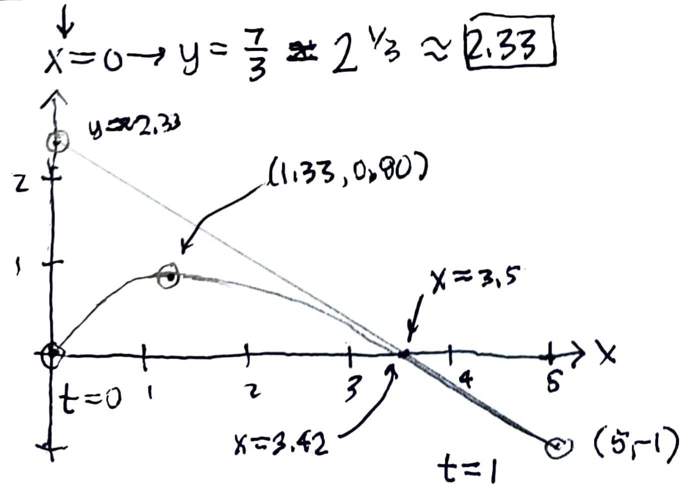
c) $\frac{dy}{dx}=0 \to y'=0 = 4-10t \to t = \frac{4}{10} = \frac{2}{5}$

$x\left(\frac{2}{5}\right) = 2\left(\frac{2}{5}\right)^3 + 3\left(\frac{2}{5}\right) = \frac{2}{5}\left(2\cdot\frac{4}{25}+3\right) = \frac{2}{5}\left(\frac{83}{25}\right)$

$= \frac{166}{125}$

$y\left(\frac{2}{5}\right) = 4\left(\frac{2}{5}\right) - 5\left(\frac{2}{5}\right)^2 = \frac{2}{5}\left(4-\cancel{5}\cdot\frac{2}{\cancel{5}}\right) = \frac{4}{5}$

$\boxed{(x,y) = \left(\frac{166}{125}, \frac{4}{5}\right) \approx (1.33, 0.80)}$



a) x-intercepts

$0 = y = 4t-5t^2 = t(4-5t)$

$t=0, \quad \frac{4}{5} = 0.8$

$x = 2t^3+3t \to 0$

$= t(2t^2+3)$

$x = \frac{4}{5}\left(2\left(\frac{16}{25}\right)+3\right)$

$= \frac{4}{5}\left(\frac{32+75}{25}\right)$

$= \frac{432}{625} \approx 3.42$