

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, IDENTIFYING expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation IF appropriate). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them, unless specifically instructed.

## pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet on top of your answer sheets as a cover page, with the first test page facing up:

"During this examination, all work has been my own. I have not accessed any of the class web pages or any other sites during the exam. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: \_\_\_\_\_

Date: \_\_\_\_\_

1. Consider the polar curve  $r = \theta^2$ ,  $\theta = 0 \dots \pi$ .

- Make a rough sketch of this curve, with appropriately labeled tickmarks (from a technology screen).
- Set up and evaluate by hand (factor and  $u$ -sub!) the integral for the arclength of this curve segment.
- Set up and evaluate by hand the integral for the area enclosed by this curve segment.

2. Consider the polar curve circle  $r = \cos(\theta) + \sin(\theta)$  shown in the figure.

- At what negative acute angle does the circle pass through the origin?
- Set up an integral to evaluate the area of the part of the circle in the fourth quadrant.

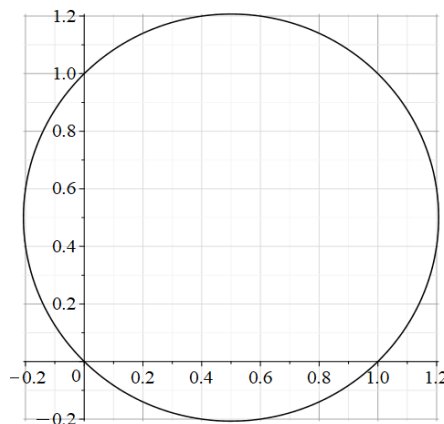
By expanding and simplifying the integrand, one finds an integral one can do by  $u$ -substitution. Evaluate this by hand.

- The rectangle which contains this part of the circle has area  $\frac{1}{5}$ . Does your result seem reasonable compared to this value?

d) Show that the Cartesian equation  $x^2 + y^2 = x + y$  converts to the polar equation  $r = \cos(\theta) + \sin(\theta)$ .

e) What are the horizontal intercepts of this circle? [Solve the Cartesian equation for them!]

f) Use the quadratic formula to solve for  $y$  for the lower semicircle and use Maple to integrate  $-y$  between the intercepts to check your area integral above.



3. Consider the curve segment:  $x = 2t^3 + 3t$ ,  $y = 4t - 5t^2$ ,  $t = 0 \dots 1$ .

- Where does this curve intersect the coordinate axes? Give the exact and 2 decimal place values of the coordinates.
- Write the equation of the tangent line at  $t = 1$ . Where does this tangent line intersect the horizontal and vertical axes?
- Find the point on the curve where the tangent line is horizontal.
- Using these various points found above as a guide, make a rough sketch of this curve segment with this tangent line (from a technology screen) with appropriately labeled tickmarks (use a straight edge for the tangent line), labeling the intercepts of the tangent line by their decimal values (2 decimal places) and labeling the horizontal tangent point by its coordinates (2 decimal places). Make your sketch large enough so all this is relatively clear.

Remember for a parametrized curve or a polar curve, use PlotBuilder selecting the corresponding option or:

```
> plot([t, t^2, t=0..1])
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> plot(cos(theta), theta=0..2*pi, coords=polar)
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Remember: " " (space) or "\*" for multiplication ALWAYS,  $\pi \neq \text{pi}$ ,  $e \neq e$ . Context menu plots can be copied and pasted together. Gridlines and 1-1 option may be useful. You can copy and paste separate plots together provided you have the same horizontal window, chosen in the PlotBuilder options. [For example, putting a tangent line on a parametrized curve plot. Ask bob for help if this gives you trouble.]

Do not put any work here!