MAT1505-05 22F Test 3 Print Name (Last, First) __________________ , ________________|____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). INDICATE where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them.

pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet on top of your answer sheets as a cover page, with the first test page facing up:
"During this examination, all work has been my own. I have not accessed any of the class web pages or any other sites during the exam. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date:

1. Consider the power series $f(x) = \sum_{n=0}^{\infty} \left(\frac{(x-1)^n}{2^{n+1} \cdot (n+1)\,(n+3)} \right)$.

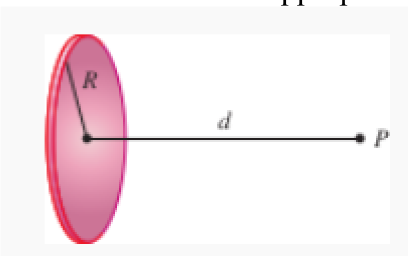
a) Evaluate its radius R of convergence.
b) Examine the endpoints to determine the full interval of convergence, and justify your conclusions.
c) When $x = 0$, this is an alternating series. For what value of n does then next term in the series have a value less than 10^{-4} ? State the term values for n and $n+1$.
d) Compare $S_n = \sum_{k=0}^{n} \left(\frac{(-1)^k}{2^{k+1} \cdot (k+1)\,(k+3)} \right)$ for that value of n with Maple's numerical value for the infinite series. Is the difference less than 10^{-4} ?
e) **Optional.**
If you are curious and you > assume($a < x$ **and** $x < b$) where (a, b) is the open interval of convergence within the full interval, Maple can actually sum this series to the original function for which it is a Taylor series, which helps explain its radius of convergence. Either this function or the above expression for the infinite series can be plotted to confirm the full interval of convergence about its center.

2. A patient is injected with a drug every 12 hours. Immediately before each injection the concentration of the drug has been reduced by 65% and the new dose increases the concentration by 1.5 mg/L . [Recall lecture notes for 11.2 on HW page.]
a) What is the concentration C_3 after three doses?
b) If C_n is the concentration after the n th dose, find a formula for C_n as a function of n .
c) What is the limiting value C_∞ of the concentration (2 decimal places)?
d) What is the first (integer!) value of n when C_n reaches within 1 percent of C_∞ ?
e) What is the minimum level of the concentration before each injection that is reached asymptotically (2 decimal places)?
[Clearly this minimal level for effectiveness of the drug is important!]

3. A uniformly charged disk has radius R and surface charge density σ as in the figure. The electric potential V and at a point P at a distance d along the perpendicular central axis of the disk is

$$V = 2\pi k_e \sigma \cdot \left(\sqrt{d^2 + R^2} - d\right).$$

a) Show that $V \approx \dfrac{\pi k_e R^2 \sigma}{d}$ using the binomial series expansion in the small ratio $\dfrac{R}{d}$ assuming it to be much smaller than 1 appropriate for distances d large compared to R.



b) Use the alternating series error estimate to show that this approximation is accurate to within 1 percent as long as $d > 5R$.

c) Confirm this estimate by evaluating the true percentage error when $d = 5R$

$$: \ ErrorExact = \frac{approx - exact}{exact} \cdot 100 .$$

Put all work and answers on separate sheets of paper.