

① a)  $f(x) = x \sin \pi x$

$$A = \int_0^1 x \sin \pi x \, dx = \frac{\sin \pi x - x \pi \cos \pi x}{\pi^2} \Big|_0^1$$

$$= \frac{\sin \pi - \pi \cos \pi - \sin 0 - 0 \cdot \pi \cos 0}{\pi^2} = \frac{1}{\pi}$$

$p(x) = \pi x \sin \pi x$

$$\mu = \int_0^1 x p(x) \, dx = \int_0^1 x (\pi x \sin \pi x) \, dx = \int_0^1 \pi x^2 \sin \pi x \, dx$$

$$= \frac{-\pi^2 x^2 \cos \pi x + 2 \cos \pi x + 2 \pi x \sin \pi x}{\pi^2} \Big|_0^1$$

$$= \frac{-\pi^2 \cos \pi + 2 \cos \pi + 2 \pi \sin \pi - (-0 + 2(1) + 0)}{\pi^2}$$

$$= \frac{\pi^2 - 4}{\pi^2} \approx 0.59472 \approx \boxed{0.595}$$

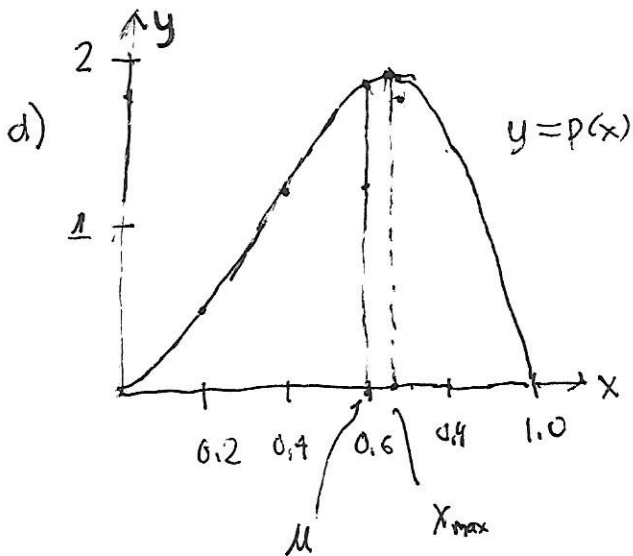
d)  $0 = f'(x) = \frac{d}{dx} (x \sin \pi x) = 1 \cdot \sin \pi x + x (\cos \pi x) (\pi)$

$$\sin \pi x + \pi x \cos \pi x = 0 \quad \text{or}$$

$$\tan \pi x = -\pi x \quad \text{on } [0, 1].$$

↓  
Solve Numerical  
from point  $x = 0.5$ :

$$x \approx 0.64577 \approx \boxed{0.646} \approx x_{\max}$$



b)  $P(\frac{1}{2} \leq x \leq 1) = \int_{\frac{1}{2}}^1 p(x) \, dx$

$$= \int_{\frac{1}{2}}^1 \pi x \sin \pi x \, dx$$

$$= \frac{\sin \pi x - \pi x \cos \pi x}{\pi^2} \Big|_{\frac{1}{2}}^1$$

$$= \frac{\sin \pi - \pi \cos \pi - \sin \pi/2 - \pi/2 \cos \pi/2}{\pi^2}$$

$$= \frac{\pi - 1}{\pi} \approx 0.68169 \approx \boxed{0.682}$$

