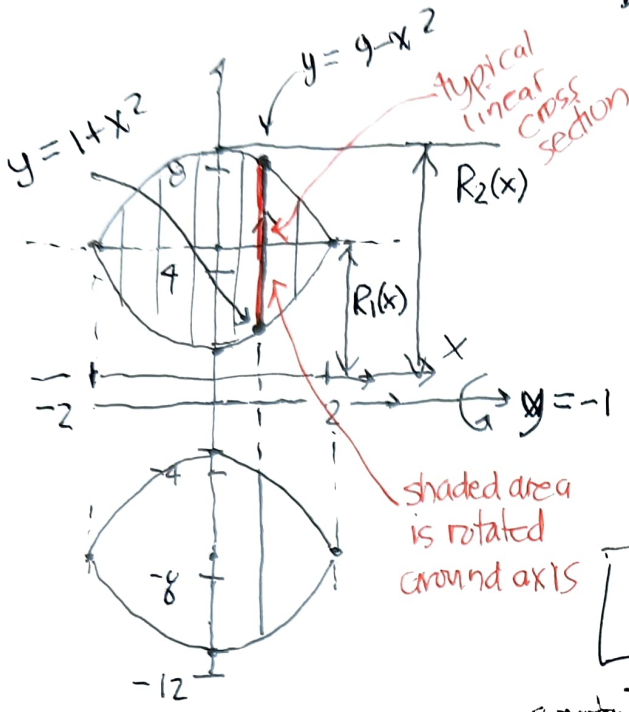
MAT1505-05 22F Test 1 Answers

① a) $y = 9 - x^2$, $y = 1 + x^2$ → $9 - x^2 = 1 + x^2$, $8 = 2x^2$, $x^2 = 4$, $x = \pm 2$

$y = 1 + 4 = 5$

intersection points must be donwn!



b) $R_1(x) = 1 + x^2 - (-1) = 2 + x^2$

$R_2(x) = 9 - x^2 - (-1) = 10 - x^2$

} differences of functions with −1

$A(x) = \pi(R_2(x)^2 - R_1(x)^2)$

$= \pi((10 - x^2)^2 - (2 + x^2)^2]$

$= \pi[100 - 20x^2 + x^4 - (4 + 4x^2 + x^4)]$

$= \pi[96 - 24x^2]$ always simplify.

$V = \int_{-2}^{2} A(x)\,dx$

$= \int_{-2}^{2} \pi[96 - 24x^2]\,dx$

c) $V = \pi\left[96x - 24\frac{x^3}{3}\right]_{-2}^{2} \overset{\text{Symmetry}}{=} 2\pi\left[96x - 8x^3\right]_{0}^{2} = 2\pi[192 - 64]$

$= \boxed{256\pi} \approx 804.24772$

d) Maple agrees.

② a) $V_{rms}^2 = V_0^2 \int_0^{1/60} (60)\sin^2(120\pi t)\,dt$ ($u = 120\pi t$, $dt = \frac{du}{120\pi}$)

$u = 120\pi t$

$du = 120\pi\,dt$

$dt = \frac{du}{120\pi}$

$t = 0 \to u = 0$

$t = \frac{1}{60} \to u = \frac{120\pi}{60} = 2\pi$

$= \frac{V_0^2}{2\pi} \int_0^{2\pi} \sin^2 u\,du = V_0^2 \left[\frac{\int_0^{2\pi} \sin^2 u\,du}{2\pi}\right] \overset{\text{Maple}}{=} V_0^2\left[\frac{\pi}{2\pi}\right] = \frac{1}{2}V_0^2$

$C = 1/2$

b) $V_0 = \frac{120}{\sqrt{C}} = \frac{120}{\sqrt{1/2}} = \boxed{120\sqrt{2} \approx 169.7}$ volts!

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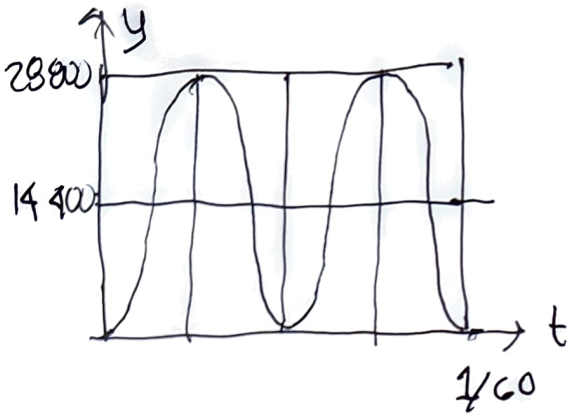
② c) $V^2 = V_0^2 \sin^2(120\pi t) \approx 28800 \sin^2(377t)$

$(120\sqrt{2})^2 = 2 \cdot 120^2$



MAT 2505-05 Test 1 22F Answers (3)

(3) a) $r(t) = 110 + 15\ln(3t+1)$ $\quad 0 \le t \le 10$

$= B'(t)$ = time rate of change of number of beats

$$\int_0^{10} r(t)\,dt = \int_0^{10} B'(t)\,dt = B(t)\Big|_0^{10} = B(10) - B(0)$$

This is the total number of beats in 10 minutes.

b) $$\int_0^{10} 110 + 15\ln(3t+1)\,dt \underset{\text{Maple}}{=} 950 + 155\ln 31$$

$$\approx 1482.3 \approx \boxed{1482} \text{ beats}$$

c) $\int 110 + 15\ln(3t+1)\,dt$

$= 110t + 15\int \ln(3t+1)\,dt$ $\quad = \frac{dw}{3}$

$w = 3t+1$
$dw = 3\,dw$
$dt = dw/3$

$$15\int \ln w \frac{dw}{3} = 5\int \underbrace{\ln w}_{u}\underbrace{dw}_{dv} = 5\left[w\ln w - \underbrace{\int w \frac{dw}{w}}_{w}\right] + C$$

$= 5(w(\ln w - 1)) + C$ $\quad u = \ln w \quad dv = dw$

$du = \frac{dw}{w} \quad v = w$

$= 5(3t+1)[\ln(3t+1) - 1] + C$

$$= \boxed{110t + 5(3t+1)[\ln(3t+1) - 1] + C}$$ antiderivative specifically requested!

$$\int_0^{10} 110 + 15(3t+1)\,dt = \left[110t + 5(3t+1)[\ln(3t+1) - 1]\right]\Big|_0^{10}$$

$= 1100 + \underbrace{5(31)}_{155}(\ln 31 - 1) = 950 + 155\ln 31$

$\approx \boxed{1482.3}$

$\approx \boxed{1482}$ agrees!