MAT1505-05 22F Test 1   Print Name (Last, First) ___________________ , ________________|____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, IDENTIFYING expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them.

pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet in on top of your answer sheets as a cover page, with the first test page facing up:
"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date:

1. Consider the region of the plane between the curves $y = 9 - x^2$ and $y = 1 + x^2$ and evaluate the volume of the solid obtained by rotating that region around the axis $y = -1$ (a donut with sharp rims!) by following these steps:
a) Sketch a graph illustrating this region of the plane, the axis, and the reflected region across this axis resulting from it revolution around the given axis of rotation, and labeling clearly the two relevant radii of the circles in a typical linear cross-section needed for evaluating the plane cross-sectional area.
b) Write down the integral for the volume of the solid and simplify the integrand.
c) Evaluate step by step the integral exactly and numerically to 5 decimal places.
d) Check your integral using Maple.

2. a) The root mean square voltage $V_{rms} = 120$ volts in an 60 cycle per second AC voltage source is the square root of the average value of the time varying voltage $V = V_0 \sin(120\,\pi\,t)$ over one cycle $T = \frac{1}{60}$ sec (the period!), where V_0 is the peak voltage. **Evaluate** the average value of the square of the voltage over one period:

$$V_{rms}^{\ 2} = \frac{1}{\frac{1}{60}} \int_0^{\frac{1}{60}} V^2\, \mathrm{d}t = V_0^{\ 2} \int_{\mathbf{0}}^{\frac{\mathbf{1}}{\mathbf{60}}} \mathbf{60\, \sin^2(120\, \pi t)\, dt} \equiv c\, V_0^{\ 2}$$

in two steps. First change the variable in the boldface integral to the dimensionless time variable $u = 120\,\pi\,t$ (called the phase angle of the oscillation) and then use Maple to evaluate exactly the value of that dimensionless integral whose value is c.

b) What is the value of the peak voltage $V_0 = \frac{V_{rms}}{\sqrt{c}} = \frac{120}{\sqrt{c}}$ in volts exactly and approximately to 4 significant figures?

c) Make a rough sketch of a technology plot of the square of the voltage over one cycle with a horizontal line for the square $V_{rms}^{\ 2} = 120^2$ of the root mean square voltage. [Label everything: axes, tickmarks, etc.]

d) Nothing to do here but read this explanation: Since the power delivered to a device is proportional to the square of the voltage, the equivalent DC constant voltage power delivered by a battery equals the average power delivered by the AC power source.

[turn over!]

3. a) Suppose $r(t) = 110 + 15\ln(3t + 1)$, $0 \le t \le 10$ models the heart beat rate per minute during the first 10 minutes of exercise. What does $\int_0^{10} r(t)\,dt$ represent?

b) What is the value of this definite integral to the nearest whole number? Use Maple to evaluate this integral directly.

c) Now by hand find an antiderivative of $r(t)$ using an obvious variable substitution first to then evaluate the antiderivative of the second term in $r(t)$ using integration by parts. Then evaluate this at the given limits to evaluate the definite integral. Does it agree with part b)?

▶ solution