MAT2500-01/03 21S Quiz 9 Print Name (Last, First) ____________________

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them.

$$\int_0^2\int_1^{1+y^3}\int_0^{y^2} f(x,y,z)\,dz\,dx\,dy$$

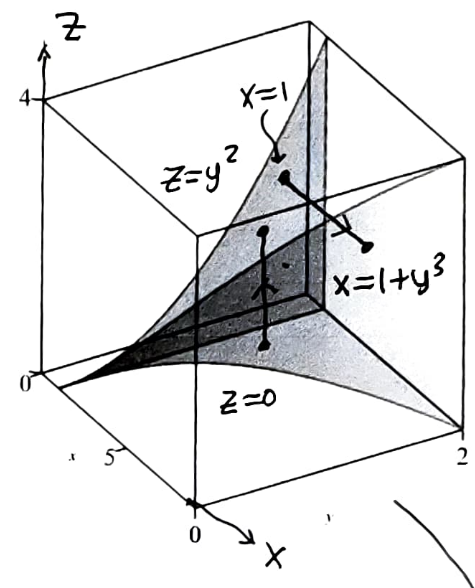
a) Evaluate this integral by hand step by step first for $f(x,y,z)=z$ and then using technology for $f(x,y,z)=1$ and then evaluate their ratio to get the z-component of the centroid. Does its location seem reasonable? Can you say why in a few words?

b) Indicate in the diagram to the right a typical labeled bullet point terminated linear cross-section for the innermost integral, and draw a completely labeled plane diagram illustrating the outer double integral.

c) Now repeat this step for integrating in the order ∭...dx dy dz .

d) Rewrite these integrals as equivalent integrals in the order ∭... dx dy dz .

e) Evaluate your two new integrals using technology. Do they agree with your previous results?



▶ solution

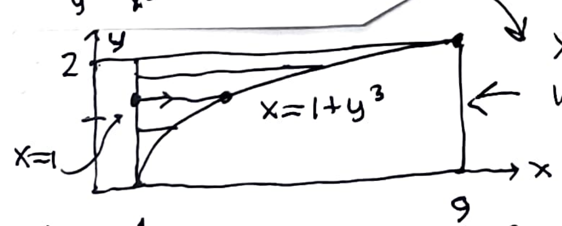
a) $$\int_0^2\int_1^{1+y^3}\underbrace{\int_0^{y^2} z\,dz}_{\frac{z^2}{2}\Big|_0^{y^2}=\frac12 y^4}\,dx\,dy = \frac12\frac{y^8}{8}\Big|_0^2 = \frac{2^8}{2^4} = 2^4 = \boxed{16}$$

$$\frac12 y^4 x\Big|_1^{1+y^3} = \frac{y^4}{2}\left[1+y^3-1\right] = \frac{y^7}{2}$$

$$\int_0^2\int_1^{1+y^3}\int_0^{y^2} 1\,dz\,dx\,dy \underset{\text{Maple}}{=} \boxed{\frac{32}{3}}$$

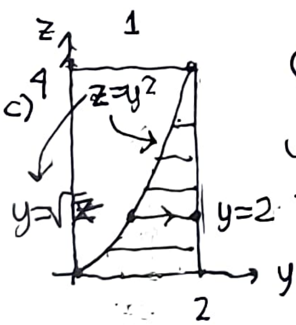
$$\bar z = \frac{16}{32/3} = \boxed{\frac32 = 1.5}$$

b) $$\int_{y=0}^{y=2}\int_{x=1}^{x=1+y^3}\cdots\,dx\,dy$$



$x=1..1+y^3$
while $y=0..2$

c)



e) $$\underbrace{\int_0^4\int_{\sqrt z}^2}\int_1^{1+y^3}\langle 1, z\rangle\,dx\,dy\,dz = \left\langle 16, \frac{32}{3}\right\rangle$$ ✓ yay!

$y=\sqrt z..2$
while $z=0..4$

Imagine looking down the x-axis of 3-d diagram to "see" the projection onto the y-z plane (keep z vertical!)