MAT2500-01/03 21S Quiz 6 Print Name (Last, First) _____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation if appropriate). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them.

1. The wind-chill index is modeled by the function $W = 13.12 + 0.6215\,T - 11.37\,v^{0.16} + 0.3965\,T v^{0.16}$ where T is the temperature (in °C) and v is the wind speed (in km/hr) . The wind speed is measured as 26 km/h, with a possible error of ± 2 km/h, and the temperature is measured as -11°C, with a possible error of ±1°C. First evaluate the differential of W at the point $(-11, 26)$. Then use differentials to estimate the maximum error in the calculated value of W due to the measurement errors in T and v. Remember the triangle inequality.

2. a) dr bob weighs about 77 kg and is around 1.81 m (although his license says 6'1" :-)). The body mass index is $B(m, h) = \frac{m}{h^2}$. What is the linear approximation of this function at the values dr bob has (to interpret small changes in his body mass index)?
b) If his weight increases to 79 kg and his height continues to shrink with old age to 1.80 m, what will his new body mass index be approximately (using the linear approximation).
c) How does the linear approximation compare to the exact value, namely what is the percentage error in using the linear approximation?

▶ solution

(1) $W = 13.12 + 0.6215\,T - 11.37\,V^{0.16} + 0.3965\,T\,V^{0.16}$

$dW = 0.6215\,dT + 0.3965\,dT\,V^{0.16} + 0.16\,V^{-0.84}(-11.37 + 0.3965\,T)\,dV$

$= (0.6215 + 0.3965\,V^{0.16})\,dT + 0.16\,(-11.37 + 0.3965\,T)\,V^{-0.84}\,dV$

$dW(-11,26) = [0.6215 + 0.3965(26)^{0.16}]\,dT + 0.16\,(-11.37 + 0.3965(-11))\,26^{-0.84}\,dV$

$\underset{\text{Maple}}{=} 1.2892885\,dT - 0.16304689\,dV$

$|dW(-11,26)| \le 1.2892885\,|dT| + 0.16304689\,|dV| \le 1.6154$

($|dT| \le 2$; triangle inequality)

max error ≈ 1.615°C

(see next page)

Mat2500-01/03 215 Quiz 6

② a) $B = mh^{-2}$ — $\frac{\partial B}{\partial m} = h^{-2}$

$\frac{\partial B}{\partial h} = -2mh^{-3}$

$(m,h) = (77, 1.81)$

$B(77,1.81) = \frac{77}{(1.81)^2} = 23.504 \approx 23.5$

$B_m(77,1.81) = (1.81)^{-2} = 0.30524$

$B_h(77,1.81) = -\frac{2(77)}{(1.81)^3} = -25.9708$

$L_B(m,h) = B(77,1.81) + B_m(77,1.81)(m-77) + B_h(77,1.81)(h-1.81)$

$= 23.504 + 0.30524(m-77) - 25.9708(h-1.81)$

$L_B(79,1.80) = 23.504 + 0.30524\underbrace{(79-77)}_{2} - 25.9708\underbrace{(1.80-1.81)}_{-0.01}$

$= 24.3737 \approx 24.374 \approx \boxed{24.4}$

Compare with $L(79,180) = 24.383$

b) $\frac{L_B(79,1.80) - L(79,1.80)}{L(79,1.80)} = 0.00037$ (Maple)

$\approx \boxed{0.04\%}$ (too high)

Realistically the body mass index is only used to the nearest integer anyway to give a "ball park" estimate for healthcare considerations, so keeping even one decimal place seems a bit exaggerated.