MAT2500-01/03 21s Quiz 5 Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation if appropriate). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them.

1. The temperature-humidity index I (or humidex, for short) is the perceived air temperature when the actual temperature is T and the relative humidity is h, so we can write $I = I(T, h)$. The following table of values of I is an excerpt from a table compiled by the National Oceanic & Atmospheric Administration. [Remember units in your responses.]

Apparent temperature as a function of temperature and humidity

Relative humidity (%)

Actual temperature (°F) T \ h	20	30	40	50	60	70
80	77	78	79	81	82	83
85	82	84	86	88	90	93
90	87	90	93	96	100	106
95	93	96	101	107	114	124
100	99	104	110	120	132	144

a) Evaluate the tabular partial derivatives $\frac{\partial I}{\partial T}(95, 60)$ and $\frac{\partial I}{\partial h}(95, 60)$

b) Using this result, by how much would you expect the perceived temperature of 114° F to decrease if the humidity decreases from 60% to 59% at an actual temperature of 95° F? By how much would you expect the perceived temperature to change if the actual temperature increases by 2° F?

2. Evaluate the 2 partial derivatives of the polynomial $f(x, y) = 2x^4y^2 - 5xy^3 + 4y$ and their values at $(x, y) = (-1, 2)$.

▶ **solution**

① a) $\Delta h = 10$

$\Delta T = 5$

	50	60	70
90		100	
95	107	114	124
100		132	

$\Delta I = 114 - 100 = 14$

$\Delta I = 132 - 114 = 18$

$\frac{\Delta I}{\Delta T} = \frac{14}{5}$, $\frac{\Delta I}{\Delta T} = \frac{18}{5}$ $> \left(\frac{\Delta I}{\Delta T}\right)_{avg} = \frac{1}{2}\,\frac{14+18}{5} = \frac{32}{10} = 3.2$

$\boxed{\left(\frac{\partial I}{\partial T}\right)(95,60) = 3.2}$ $\frac{°F}{\cancel{\%}pt}$

$\Delta I = 114 - 107 = 7$

$\Delta I = 124 - 114 = 10$

$\frac{\Delta I}{\Delta h} = \frac{10}{10} =$, $\frac{\Delta I}{\Delta h} = \frac{7}{10}$ $> \left(\frac{\Delta I}{\Delta h}\right)_{avg} = \frac{1}{2}\left(\frac{10+7}{10}\right) = \frac{1.7}{2} = 0.85$

$\boxed{\left(\frac{\partial I}{\partial h}\right)(95,60) = 0.85}$ $\frac{°F}{°F}$

b) $\Delta h = 59 - 60 = -1$

$\Delta I = \left(\frac{\partial I}{\partial h}\right)(95,60)\,\Delta h = 0.85(-1) = \boxed{-0.85\ °F} \to$ decrease by 0.85 °F

$\Delta I = \left(\frac{\partial I}{\partial T}\right)(95,60)\,\Delta T = 3.2\ (2) = \boxed{6.4\ °F} \to$ increase by 6.4 °F

② $f(x,y) = 2x^4y^2 - 5xy^3 + 4y$

$f_x(x,y) = \frac{\partial}{\partial x}(2x^4y^2 - 5xy^3 + 4y) = 2y^2\frac{\partial}{\partial x}x^4 - 5y^3\frac{\partial}{\partial x}x + 0 = 2y^2(4x^3) - 5y^3 = \boxed{8x^3y^2 - 5y^3}$

$f_y(x,y) = \frac{\partial}{\partial y}(2x^4y^2 - 5xy^3 + 4y) = 2x^4\frac{\partial}{\partial y}y^2 - 5x\frac{\partial}{\partial y}y^3 + 4 = 2x^4(2y) - 5x(3y^2) + 4$
$= \boxed{4x^4y - 15xy^2 + 4}$

$f_x(-1,2) = 8(-1)^3 2^2 - 5\cdot 2^3 = -32 - 40 = \boxed{-72}$

$f_y(-1,2) = 4(-1)^4\cdot 2 - 15(-1)\cdot 2^2 + 4 = 8 + 60 + 4 = \boxed{72}$