MAT2500-01/03 21S Quiz 1 Print Name (Last, First) ______________________

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not subsitute for them.

Given the two spheres described by the equations:

$S1 : x^2 + y^2 + z^2 + 2\,y + 2\,z + 1 = 0,\ S2 : x^2 + y^2 + z^2 - 4\,x - 4\,y + 4 = 0$

a) Complete the squares to find the coordinates of their centers and their radii: $(C1, r1)$, $(C2, r2)$.

b) Evaluate the distance d between the centers. [As a check, the fractional part of its numerical value is 0.74 to 2 decimal places.]

c) Do the two spheres overlap or not (compare d with their radii)?

d) Make a rough hand diagram of the cross-section plane through the symmetry axis connecting the centers of these spheres by plotting a circle of radius $r1$ at the origin of the x-y plane and a circle of radius $r2$ at a distance d along the positive x-axis. Include unit tickmarks on each axis. Be sure to label the points on the x-axis where the two circles intersect that axis with their numerical values to 2 decimal places when not integers. Does this diagram confirm your conclusion to part c)?

e) Derive the most simplified form of the equation for the plane of points which are equidistant from the two centers.

▶ solution

a) $x^2 + y^2 + 2y + z^2 + 2z + 1 = x^2 + (y+1)^2 - 1 + (z+1)^2 - 1 + 1 = x^2 + (y+1)^2 + (z+1)^2 - 1$

$x^2 - 4x + y^2 - 4y + z^2 + 4 = (x-2)^2 - 4 + (y-2)^2 - 4 + z^2 + 4 = (x-2)^2 + (y-2)^2 + z^2 - 4$

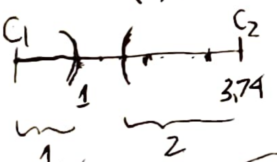
$(x-0)^2 + (y-(-1))^2 + (z-(-1))^2 = 1$

$(x-2)^2 + (y-2)^2 + (z-0)^2 = 2^2$

$$\boxed{\begin{array}{ll} C_1(0,-1,-1) & r_1 = 1 \\ C_2(2,2,0) & r_2 = 2 \end{array}}$$

b) $\overrightarrow{C_1C_2} = \langle 2,2,0\rangle - \langle 0,-1,-1\rangle = \langle 2,3,1\rangle$

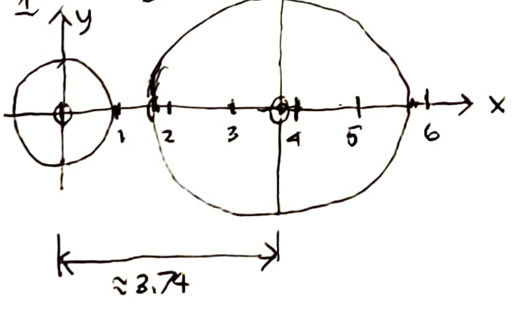
$|\overrightarrow{C_1C_2}| = \sqrt{4+9+1} = \boxed{\sqrt{14} = d} \approx 3.74166 \approx 3.74$ ✓

c) 

clearly they dont overlap since $r_1 + r_2 = 3 < d \approx 3.74$

$\left(\begin{array}{c} d - r_1 - r_2 \approx 0.74 \\ \text{separation of spheres} \end{array}\right)$

$3.74 - 2.00 = 1.74$

d) 

e) $x^2 + (y+1)^2 + (z+1)^2 = (x-2)^2 + (y-2)^2 + z^2$

$x^2 + y^2 + 2y + 1 + z^2 + 2z + 1 = x^2 - 4x + 4 + y^2 - 4y + 4 + z^2$

$2y + 2z + 1 = -4x + 4 - 4y + 4 \rightarrow 4x + 6y + 2z = 8 - 2 = 6 \rightarrow \boxed{2x + 3y + z = 3}$