MAT1505-01/02 21F Quiz 8 Print Name (Last, First) ________________, ________________|____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

No technology is needed today. Use words to explain, show appropriate algebra and limits or limiting behavior.

1. Is $\sum_{n=1}^{\infty}(-1)^n \cdot \sqrt{n}\,\sin\left(\frac{1}{n}\right)$ absolutely convergent, conditionally convergent or divergent? Justify your claim.

2. Is $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(n+1)\,3^n}{2^{2n+1}}$ absolutely convergent, conditionally convergent or divergent? Justify your claim.

3. Is $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{e^{\frac{1}{n}}}{\sqrt{2+n}}$ absolutely convergent, conditionally convergent or divergent? Justify your claim.

▶ solution

① $|a_n| = \sqrt{n}\,\underbrace{\sin\frac{1}{n}}_{\to 0} \xrightarrow{n\to\infty} n^{1/2}\left(\frac{1}{n}\right)$ since $\lim_{x\to 0}\frac{\sin x}{x} = 1$ so $\sin x \approx x$ for small x

$= \frac{1}{n^{1/2}} = b_n$

[so $\lim_{n\to\infty}\frac{|a_n|}{b_n} = 1$ same convergence properties]

so $\sum_{n\to\infty} a_n \underset{\text{"behaves like"}}{\sim} \sum_{n=1}^{\infty}(-1)^n\frac{1}{n^{1/2}}$ alternating $p=\frac{1}{2}$ series converges conditionally since the $p=\frac{1}{2}$ series diverges $p \le 1$

so original series $\boxed{\text{converges conditionally}}$

② $\left|\frac{a_{n+1}}{a_n}\right| = \frac{(n+2)\,3^{n+1}}{2^{2(n+1)+1}} \cdot \frac{2^{2n+1}}{(n+1)\,3^n} = \left(\frac{n+2}{n+1}\right)\cdot\frac{2^{2n+1}}{2^{2n+3}}\cdot(3)$

$= \underbrace{\left(\frac{n+2}{n+1}\right)}_{\to 1}\left(\frac{3}{4}\right) \xrightarrow{n\to\infty} \frac{3}{4} < 1$ $\boxed{\text{absolutely convergent}}$ by abs. conv. ratio test

③ $\frac{e^{\frac{1}{n}}}{\sqrt{2+n}} \xrightarrow{n\to\infty} \frac{e^0}{n^{1/2}} = \frac{1}{n^{1/2}}$ so looks like $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^{1/2}}$

which $\boxed{\text{converges conditionally}}$ just like in ①.