MAT1505-01/02 21F Test 1 Print Name (Last, First) _____ , _____|____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet in on top of your answer sheets as a cover page, with the first test page facing up:
"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date:

1. Consider the region of the plane between the curves $y = x^2$ and $y = \sqrt{x}$ and evaluate the volume of the solid obtained by rotating that region around the axis $y = 1$ by following these steps:
a) Sketch a graph illustrating this region of the plane, the axis, and the reflected region across this axis resulting from it revolution around the given axis of rotation, and labeling clearly the two relevant radii of the circles in a typical linear cross-section needed for evaluating the plane cross-sectional area.
b) Write down the integral for the volume of the solid and simplify the integrand.
c) Evaluate step by step the integral exactly and numerically to 5 decimal places.
d) Check your integral using Maple.

2. Water flows from the bottom of a storage tank at a rate of $r(t) = 200 - 4t$ liters per minute, where $0 \le t \le 50$. Remember to include units in your answers.
a) What happens at $t = 50$?
b) Find the amount of water that flows from the tank during the first 10 minutes.
c) Find the amount of water $W(t)$ that flows from the tank during the first t minutes. What is the value of $W(50)$?
d) What is the average amount of water *Wavg* that has flowed out during this time interval $0 \le t \le 50$? What is the interpretation of this quantity (given its units)?
e) Sketch a plot of $W(t)$ and the constant functions *Wavg* and $W(50)$ together over this interval (label axes and tickmarks).

3. a) Perform the rescaling change of variable $w = \frac{x}{a}$ (where $a > 0$) in the integral $A_{peak} = \int_0^a x\, e^{-\frac{x^2}{2a^2}}\, dx$ which represents the area under this curve up to the peak value of $f(x) = x\, e^{-\frac{x^2}{2a^2}}$ to obtain a purely numerical integral coefficient times some power of a.
b) Evaluate the transformed numerical integral step by step.
c) What is the average value of this function f on this interval $0 \le x \le a$?
d) For $a = 1$ sketch a graph of the curve and its average value on this interval. Does it look right?
e) **Optional.** For this value of a, at what value of x does the function equal the average value? [requires numerical solution]

▶ solution