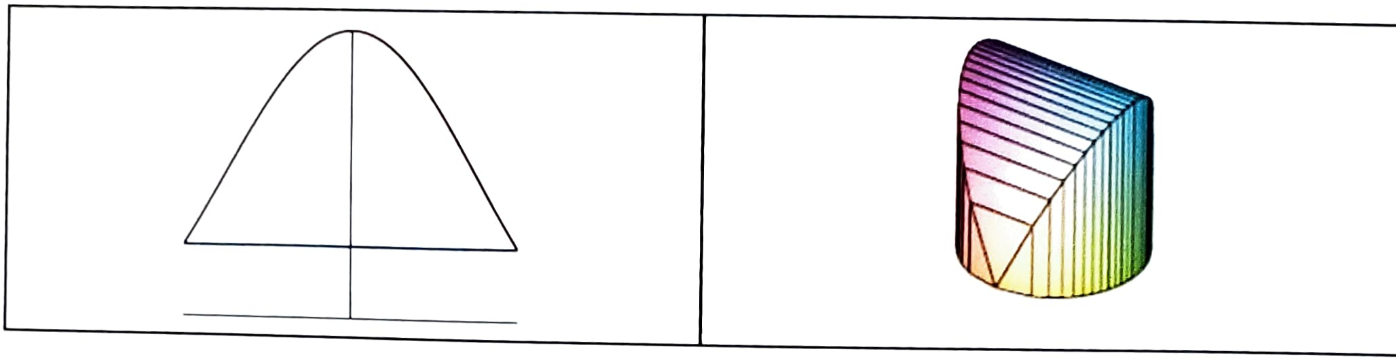
MAT1505-01/02 21F Quiz 3 Print Name (Last, First) ______________, ______________

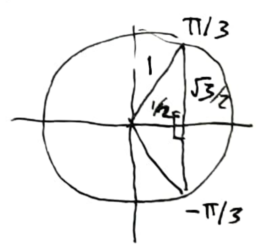
Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). INDICATE where technology is used and what type (Maple, GC).



1. a) Identify the interval around the origin where $\cos^2(x) \geq \frac{1}{4}$ and rotate the corresponding region shown in the left figure about the x-axis.
b) Draw a diagram with this region shaded by equally spaced linear cross-sections, labeling the bullet point endpoints of a typical such cross-section, and indicating the relevant radii for revolving it around the x-axis, while showing the reflection of this region across the axis of revolution.
c) Set up the integral for the volume of this solid, and then
d) use Maple to evaluate it exactly and then numerically to 4 decimal place accuracy.

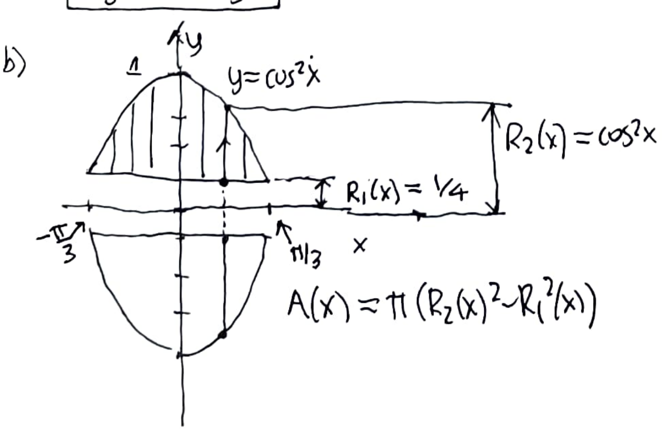
2. The base of the solid region S is a circle of radius r. Parallel plane cross-sections perpendicular to the base are squares as shown in the right figure.
a) To justify your integral for the volume of this solid, set up a labeled diagram of the region of the x-y plane with the circular base centered at the origin, labeling a typical linear cross-section with the side length used to evaluate the plane cross-section area.
b) Write down the integral needed to evaluate the volume.
c) Evaluate it by hand exactly and then to 4 decimal place accuracy.
d) Guess what fraction of the volume this represents compared to the obvious cylinder which encloses this solid, then evaluate it numerically.

1 a) $\cos^2 x = \frac{1}{4} \to \cos x = \pm\frac{1}{2} \to x = \arccos\left(\pm\frac{1}{2}\right) = \pm\arccos\frac{1}{2} = \pm\pi/3$



$\boxed{-\frac{\pi}{3} \leq x \leq \frac{\pi}{3}}$

b)



$A(x) = \pi\left(R_2(x)^2 - R_1^2(x)\right)$

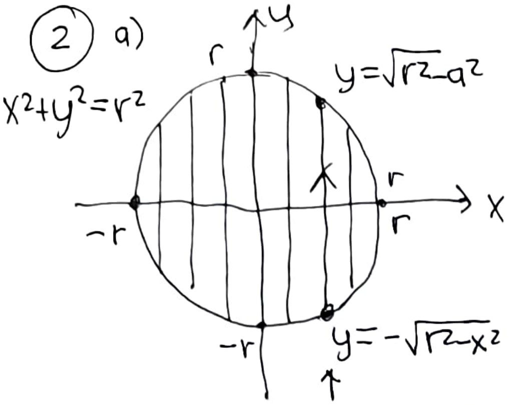
c) $V = \int_{-\pi/3}^{\pi/3} \pi\left((\cos^2(x))^2 - (1/4)^2\right) dx$

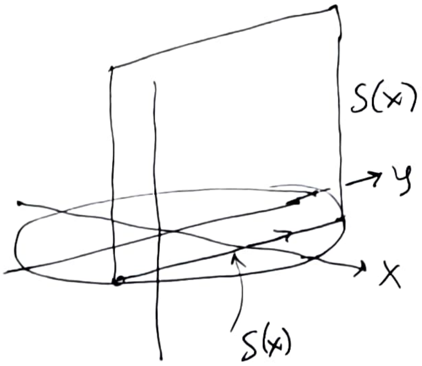
$= \boxed{\int_{-\pi/3}^{\pi/3} \pi\left(\cos^4 x - 1/16\right) dx}$

$\underset{\text{Maple}}{=} \boxed{\frac{5\pi^2}{24} + \frac{7\pi\sqrt{3}}{32} \approx 3.2464}$

MAT1505-01/02 21F Quiz 3

2 a)





cross-section length $S(x)=2\sqrt{r^2-x}$

$$A(x) = S(x)^2 = 4(r^2-x^2)$$

either:

b) $$V = \int_{-r}^{r} A(x)\,dx = 2\int_0^r A(x)\,dx = \boxed{2\int_0^r 4(r^2-x^2)\,dx} = \boxed{\int_{-r}^{r} 4(r^2-x^2)\,dx}$$

c) $$= 8\left(r^2x-\frac{x^3}{3}\right)\Big|_0^r = 8\left(r^3-\frac{r^3}{3}\right) = 8\left(\frac{2}{3}r^3\right) = \boxed{\frac{16r^3}{3}}$$

$$\boxed{\approx 5.3333\ r^3}$$

Maple

d) max square side when $x=0 \to s=2r$, height of cylinder

$$V_{cyl} = \underbrace{(\pi r^2)}_{A}\ \underbrace{(2r)}_{h} = 2\pi r^3$$

$$\frac{V}{V_{cyl}} = \frac{16r^3/3}{2\pi r^3} = \frac{8}{3\pi} \approx 0.849 \approx 85\%$$

Looking at the plot, this fraction has to lie between ½ and 1 but clearly closer to 1 so between 3/4 & 1, so $\gtrsim 75\%$, but no way can we guess much better.