MAT2500-01/02 20S Quiz 7 Print Name (Last, First) ______________________________

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them.

1. Consider the level surface $F(x,y,z) = z - x^2 - y^2 = 1$ at the point (1,2,6).

a) Write the equation for the tangent plane there simplifying it to the standard linear form, then solving for z.

b) Write a parametrized vector equation for the normal line at the point (1,2,6). At what point does this line intersect the z axis?

c) Evaluate the directional derivative of F at (1,2,6) towards the point (3,4,4) and approximate it to 3 decimal places.

2. $f(x,y) = 4xy^2 - x^2y^2 - xy^3$.

Evaluate $f_{xx}f_{yy} - f_{xy}^2$

▶ solution

1 a) $F(x,y,z) = z - x^2 - y^2 = 1$

$\vec\nabla F(x,y,z) = \langle \frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z} \rangle$

$= \langle -2x, -2y, 1 \rangle$

$\vec\nabla F(1,2,6) = \langle -2, -4, 1 \rangle = \vec n$

$\vec r_0 = \langle 1,2,6 \rangle$

$0 = \vec n \cdot (\vec r - \vec r_0) = \langle -2,-4,1 \rangle \cdot \langle x-1, y-2, z-6 \rangle$

$= -2(x-1) - 4(y-2) + 1(z-6)$

$= -2x - 4y + z \underbrace{+2+8-6}_{4}$

$\boxed{2x + 4y - z = -4}$

b) $\vec r = \vec r_0 + t\vec n = \langle 1,2,6 \rangle + t\langle -2,-4,1 \rangle$

$\langle x,y,z \rangle = \langle 1-2t, 2-4t, 6+t \rangle$

z-axis: $x = 0 = y \rightarrow z = 6 + \frac{1}{2} = \frac{13}{2}$

$(1-2t = 0 = 2(1-2t))$

$t = \frac{1}{2}$

z-axis intersection: $\boxed{(0,0,13/2) = (0,0,6.5)}$

c) $Q(3,4,4)$: $\vec{PQ} = \langle 3,4,4 \rangle - \langle 1,2,6 \rangle = \langle 3-1, 4-2, 4-6 \rangle$

$P(1,2,6)$: $= \langle 2,2,-2 \rangle$, $|\vec{PQ}| = 2\sqrt3$

$\hat u = \hat{PQ} = \frac{\langle 1,1,-1 \rangle}{\sqrt3}$

$D_{\hat u} f(1,2,6) = \hat u \cdot \vec\nabla f(1,2,6) = \frac{\langle 1,1,-1 \rangle}{\sqrt3} \cdot \langle -2,-4,1 \rangle$

c) (continued)

$\ldots = \frac{1(-2) + 1(-4) + (-1)(1)}{\sqrt3} = \frac{-2-4-1}{\sqrt3}$

$\boxed{= -\frac{7}{\sqrt3}} \approx -4.041451$

$\approx \boxed{-4.041}$ 3 decimal places

2 $f(x,y) = 4xy^2 - x^2y^2 - xy^3$

$f_x = 4y^2 - 2xy^2 - y^3$

$f_{xx} = -2y^2$

$f_y = 8xy - 2x^2y - 3xy^2$

$f_{yy} = 8x - 2x^2 - 6xy$

$f_{xy} = \frac{\partial}{\partial y}(4y^2 - 2xy^2 - y^3)$

$= 8y - 4xy - 3y^2$

$f_{xx}f_{yy} - f_{xy}^2 =$

$(-2y^2)(8x - 2x^2 - 6xy)$

$- (-8y + 4xy + 3y^2)^2$

$= \boxed{-16xy^2 + 4x^2y^2 + 12xy^3 - (-8y+4xy+3y^2)^2}$

Maple ... long expression not important to expand here

$= -y^2(12x^2 + 12yx + 9y^2 - 48x - 48y + 64)$