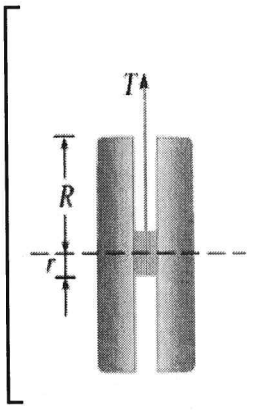
MAT2500-01/02 20S Quiz 6 Print Name (Last, First) _____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation if appropriate). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them.

1. The tension T in the string of a yo-yo whose cross-section is shown in the figure is $T = \dfrac{WR}{2r^2 + R^2}$, where W is the weight of the yo-yo.



a) Use differentials to estimate the change in the tension if R is increased from 3 cm to 3.1 cm and r is increased from 0.7 to 0.8 cm. Does the tension increase or decrease?

b) Now redo the problem to first obtain the linear approximation to T at $(R, r) = (3, 0.7)$, and then use it to evaluate the tension itself at the nearby point $(R, r) = (3.1, 0.8)$.

c) Evaluate the exact change in the tension to compare with your approximate value as a check on part a). Does this look like a good approximation?

Give your final answers to 3 significant figures (keep 4 significant figures in your intermediate numbers).

d) **Optional.** What is the percentage error in your approximate change compared to the exact change. [This quantifies how good the approximation is.]

▶ solution

① a) $T = \dfrac{WR}{2r^2+R^2} = WR(2r^2+R^2)^{-1}$

$T(3,0.7) = \dfrac{W(3)}{2(.49+9)} = 0.3006W$

$\dfrac{\partial T}{\partial R} = W\dfrac{(2r^2+R^2)(1) - R(0+2R)}{(2r^2+R^2)^2} = W\dfrac{(2r^2-R^2)}{(2r^2+R^2)^2}$

$dR = 3.1-3 = 0.1$

$dr = 0.8-0.7 = 0.1$

$\dfrac{\partial T}{\partial r} = WR(-1)(2r^2+R^2)^{-2}(4r+0) = \dfrac{-W(4)r}{(2r^2+R^2)^2}$

$dT = \dfrac{\partial T}{\partial R}dR + \dfrac{\partial T}{\partial r}dr = \dfrac{(2r^2-R^2)W\,dR - 4Wr\,dr}{(2r^2+R^2)^2}$

$dT\big|_{(R,r)=(3,0.7)} = \dfrac{2(.49-9)W\,dR - 2.8W\,dr}{(2(.49)+9)^2} \overset{\text{Maple}}{=} -0.08052W\,dR - 0.08434W\,dr$

$dT\big|_{\substack{(R,r)=(3,0.7)\\(dR,dr)=(0.1,0.1)}} = -0.08052(0.1)W - 0.08434(0.1)W \overset{\text{Maple}}{=} -0.016485W \approx \boxed{-0.0165\,W}$

b) $L(R,r) = T(3,0.7) + \dfrac{\partial T}{\partial R}(3,0.7)(R-3) + \dfrac{\partial T}{\partial r}(3,0.7)(r-0.7)$

$= 0.3006W - 0.08052(R-3) - 0.08434W(r-0.7)$

$L(3.1,0.8) = 0.3006W - 0.08052(0.1) - 0.08434W(0.1) = 0.2841W \approx \boxed{0.284\,W}$

c) $L(3.1,0.8) - L(3,0.7) \overset{\text{Maple}}{=} -0.01594W \approx \boxed{-0.0159\,W}$

comparable to -0.0165

d) $\dfrac{\text{approx}-\text{exact}}{\text{exact}} = \dfrac{-0.016485+0.01594}{-0.01594} \overset{\text{Maple}}{\approx} 0.034 \approx \boxed{3.4\%}$ error, not bad