MAT2500-01/02 20S Quiz 1 Print Name (Last, First) _____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not subsitute for them.

1.

a) Find an equation of a sphere in standard form if one of its diameters has endpoints $(5, 4, 3)$ and $(1, 6, -9)$.

b) What are the center point and radius r_1 of this sphere?

c) Find the center point and radius r_2 of the second sphere: $x^2 + y^2 + z^2 - 2x - 4y + 8z = 15$.

d) What is the distance s between the centers? Evaluate numerically r_1, r_2, s and the ratios $\frac{s}{r_1}$, $\frac{s}{r_2}$

e) Based on d), do the two spheres overlap? Explain why, using the previous numerical values. [An optional sketch with circles illustrating these numbers could be useful.]

▶ solution

a) $\vec{r}_1 = \langle 5,4,3\rangle$

$\vec{r}_2 = \langle 1,6,-9\rangle$

$\vec{r}_1 - \vec{r}_2 = \langle 5-1, 4-6, 3-(-9)\rangle$

$= \langle 4,-2,12\rangle$ (diameter)

$\frac{1}{2}(\vec{r}_1 - \vec{r}_2) = \langle 2,-1,6\rangle$ (radius vector)

$r_1 = \left|\frac{\vec{r}_1 - \vec{r}_2}{2}\right| = \sqrt{4+1+36} = \sqrt{41}$ (radius)

$(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$ (standard form)

center = midpoint of diameter linesegment

$\frac{1}{2}(\vec{r}_1 + \vec{r}_2) = \frac{1}{2}(\langle 5,4,3\rangle + \langle 1,6,-9\rangle)$

$= \frac{1}{2}(\langle 6,10,-6\rangle) = \langle 3,5,-3\rangle = (a,b,c)$

$\boxed{(x-3)^2 + (y-5)^2 + (z+3)^2 = 41}$

b) $\boxed{\text{center } (3,5,-3),\ \text{radius } r_1 = \sqrt{41}}$

c) $x^2 - 2x = (x-1)^2 - 1$

$+y^2 - 4y \quad + (y-2)^2 - 4$

$+z^2 + 8z \quad + (z+4)^2 - 16$

$= (x-1)^2 + (y-2)^2 + (z+4)^2 - 21$

$= 15$

$(x-1)^2 + (y-2)^2 + (z+4)^2 = 36 = 6^2$

$\boxed{\text{center: } (1,2,-4),\ \text{radius } r_2 = 6}$

extra step helps avoid mental error

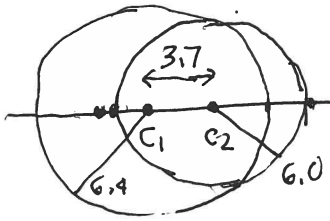
d) separation vector: $\langle 3,5,-3\rangle - \langle 1,2,-4\rangle = \langle 3-1, 5-2, -3-(-4)\rangle = \langle 2,3,1\rangle$

distance: $s = \sqrt{2^2+3^2+1^2} = \sqrt{4+9+1} = \boxed{\sqrt{14}}$.

$\boxed{s \approx 3.742,\ r_1 \approx 6.403,\ r_2 = 6.0}$

$\boxed{\frac{s}{r_1} \approx 0.584,\ \frac{s}{r_2} \approx 0.624}$

e) The separation distance is smaller than either radius so the centers are within the other sphere so they overlap.



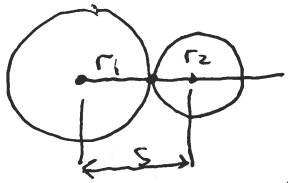
first attempt not so good.

second circle should only be a bit smaller.

The center separation is roughly half the two radii, one slightly bigger than the other.

(see Maple plot in worksheet soln)

NOTE:



$s = r_1 + r_2$ means tangent

$s < r_1 + r_2$ means overlap