MAT2500-01/02 20S Test 3 Print Name (Last, First) _____|_

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation IF appropriate). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them. Explain in as many words as possible everything you are doing! For each hand integration step, state the

antiderivative formula used before substituting limits into it: $\int_a^b f(x)\ dx = F(x)|_{x=a}^{x=b} = F(b) - F(a)$.

▼ pledge

1. $A = \int_1^e \int_0^{\ln(x)} 1\ dy\ dx$ a) Evaluate this exactly using technology and then numerically evaluate the result. [Thus avoiding an integration by parts!]

b) Make a completely labeled (shaded) diagram of the region of integration whose area A is represented by this integral, with a typical labeled cross-section representing the current iteration of the integral. This region almost looks like the triangle formed by the three corners of the region. Evaluate the area of this triangle (half base times height) and compare it to your numerical value of the integral. Do they make sense together? Explain.

c) Make a new completely labeled diagram corresponding to the reversed order of integration.

d) State the new integral with the order of integration reversed.

e) Evaluate the new integral exactly by hand and using technology.

f) Do you get the same result as in part a)? If not find your error.

2. Consider the solid region R in the first octant between the planes (sides and floor) $x = 0, y = 4, z = 0$ and the surface (ceiling)
$x^2 - y + z = 0$.

a) Corresponding to a triple integral $\iiint_R y\ dz\ dy\ dx,$

make a labeled diagram of the floor of this solid region in that plane describing the outer double integral, accompanied by a copy of the 3d diagram including an appropriate labeled bullet endpoint typical linear cross-section.

b) Write down and evaluate exactly using technology the previous iterated triple integral.

c) Now redo the previous two parts for the integration order
$\iiint_R y\ dy\ dx\ dz.$
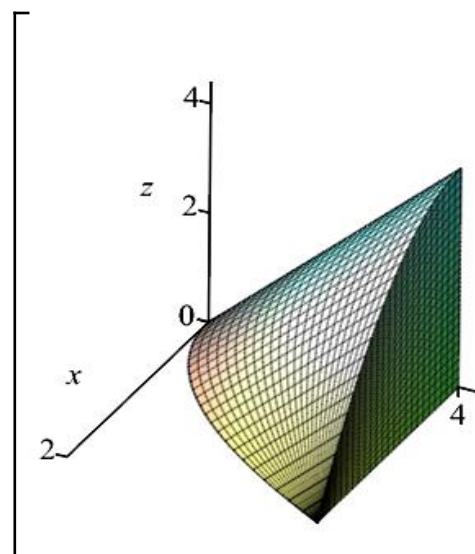
3. a) Analyze the triple integral $Q = \int_{-\sqrt{7}}^{0} \int_{0}^{\sqrt{7-x^2}} \int_{\frac{3}{\sqrt{7}}\sqrt{x^2+y^2}}^{\sqrt{16-x^2-y^2}} z \, dz \, dy \, dx$ by drawing an annotated sketch of the

outer double integral, and a rough 3-d sketch of the floor and ceiling surfaces, with the usual annotated linear cross-sections.
b) Then sketch an r-z half-plane diagram appropriate for the solid of revolution region of integration, with shading and typical annotated linear line segment for the spherical integration, and redraw the 2-d diagram appropiate for the outer double integral of the spherical integration (with the usual annotations!).
c) Re-iterate the Cartesian triple integral in spherical coordinates using your diagrams.
d) Evaluate both integrals exactly and then numerically using technology. Comment.

4. Consider solid region R consisting of the top of the sphere $x^2 + y^2 + (z-2)^2 = 4$ cut off below by the plane $z = 3$.
a) Express the two equations in cylindrical coordinates, find their intersection coordinates (r, z) and then make a diagram of the corresponding r-z half-plane shaded by equally spaced linear cross-sections corresponding to the two integration orders possible over this region (pick one), showing a typical such cross-section with bullet point endpoints labeled by the starting and stopping value equations of the corresponding integration variable. From your diagram, iterate the integral $\iiint_R f \, dV$ for any function f.

b) Now re-express your equations in spherical coordinates, and find the spherical coordinates (ρ, ϕ) of their intersection and then make the corresponding r-z half-plane diagram illustrating an iterated integral $\iiint_R f \, dV$ in

these coordinates, including labeling the starting and stopping ϕ coordinate rays, and from it, iterate this integral.
c) Evaluate the Cartesian coordinates of the centroid of this region, by first evaluating the volume and three moment integrals in cylindrical coordinates, letting Maple carry out the actual triple integral.
d) Evaluate the Cartesian coordinates of the centroid of this region, by first evaluating the volume and three moment integrals in spherical coordinates letting Maple carry out the actual triple integral.
e) Plot the centroid point in your r-z half-plane. Does it look right (considering that this is a solid of revolution around the vertical axis)? Explain.

Optional Bonus Challenge.

Explain the surprising value of the integral $\int_{0}^{\sqrt{2}} \int_{0}^{\sqrt{2-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{2-(x^2+y^2)}} 1 \, dz \, dy \, dx$ by converting to cylindrical or

spherical coordinates and analyzing the region of integration.

▶ solution