

MAT 2705-04/05 20F Quiz 5 Answers

$$\begin{aligned} 3x_1 + x_2 + x_3 + 6x_4 &= 14 \\ x_1 - 2x_2 + 5x_3 - 5x_4 &= -7 \\ 4x_1 + x_2 + 2x_3 + 7x_4 &= 17 \end{aligned}$$

a)  $\underbrace{\begin{bmatrix} 3 & 1 & 1 & 6 \\ 1 & -2 & 5 & -5 \\ 4 & 1 & 2 & 7 \end{bmatrix}}_A \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_X = \underbrace{\begin{bmatrix} 14 \\ -7 \\ 17 \end{bmatrix}}_b \rightarrow \langle A|b \rangle = C = \begin{bmatrix} 3 & 1 & 1 & 6 & 14 \\ 1 & -2 & 5 & -5 & -7 \\ 4 & 1 & 2 & 7 & 17 \end{bmatrix}$

b)  $\xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & -2 & 5 & -5 & -7 \\ 3 & 1 & 1 & 6 & 14 \\ -3 & +6 & -15 & +15 & +21 \\ 4 & 1 & 2 & 7 & 17 \\ -4 & +8 & -20 & +20 & +28 \end{bmatrix} \xrightarrow{\begin{matrix} R_2 \rightarrow R_2 - 3R_1 \\ R_3 \rightarrow R_3 - 4R_1 \end{matrix}} \begin{bmatrix} 1 & -2 & 5 & -5 & -7 \\ 0 & -7 & -14 & 21 & 35 \\ 0 & 9 & -18 & 27 & 45 \\ 0 & 9 & -18 & 27 & 45 \end{bmatrix} \xrightarrow{\begin{matrix} R_2 \rightarrow \frac{1}{7}R_2 \\ R_3 \rightarrow \frac{1}{9}R_3 \end{matrix}} \begin{bmatrix} 1 & -2 & 5 & -5 & -7 \\ 0 & 1 & -2 & 3 & 5 \\ 0 & 1 & -2 & 3 & 5 \end{bmatrix}$

$\xrightarrow{R_3 \rightarrow R_3 - R_2} \begin{bmatrix} 1 & -2 & 5 & -5 & -7 \\ +0 & +2 & -4 & +6 & +10 \\ 0 & 1 & -2 & 3 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_1 \rightarrow R_1 + 2R_2} \begin{bmatrix} 1 & 0 & 1 & 1 & 3 \\ 0 & 1 & -2 & 3 & 5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

c)  $\begin{matrix} x_1 & x_2 & x_3 & x_4 \\ \downarrow & \downarrow & & \\ \text{L} & \text{L} & \text{F} & \text{F} \end{matrix} \rightarrow \begin{matrix} \text{Leading: } \{x_1, x_2\} \\ \text{Free: } \{x_3, x_4\} \end{matrix}$

$$\begin{aligned} x_1 + x_3 + x_4 &= 3 \\ x_2 - 2x_3 + 3x_4 &= 5 \\ 0 &= 0 \end{aligned}$$

$x_3 = t_1, x_4 = t_2$

$$\begin{aligned} x_1 &= 3 - t_1 - t_2 \\ x_2 &= 5 + 2t_1 - 3t_2 \end{aligned}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 3 - t_1 - t_2 \\ 5 + 2t_1 - 3t_2 \\ t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 0 \\ 0 \end{bmatrix} + t_1 \begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \end{bmatrix} + t_2 \begin{bmatrix} -1 \\ -3 \\ 0 \\ 1 \end{bmatrix}$$

$\underbrace{\begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \end{bmatrix}}_{e_1} \quad \underbrace{\begin{bmatrix} -1 \\ -3 \\ 0 \\ 1 \end{bmatrix}}_{e_2}$

coefficient vectors of free parameters

d) Maple returns  $\begin{bmatrix} 3 - t_3 - t_4 \\ 5 + 2t_3 - 3t_4 \\ -t_3 \\ -t_4 \end{bmatrix}$

so  $\begin{bmatrix} t_1 = -t_3 \\ t_2 = -t_4 \end{bmatrix}$  (3rd variable is free)  
(4th variable is free)