MAT2705-04/05 20f Quiz 4 Print Name (Last, First) jantzen, bob

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use **proper mathematical notation**, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not subsitute for them, unless specifically requested. **Print out this quiz to respond to the graph instructions on paper.**

Suppose the number $x(t)$ (with t in months) of alligators in a swamp satisfies the differential equation

$$\frac{dx}{dt} = 0.0001\,x^2 - 0.01\,x\,,\ x(0) = x_0$$

a) What is the equilibrium population of alligators (when the derivative is zero!)

b) Use the integration formula $\displaystyle\int \frac{1}{x\cdot(x-a)}\,dx = \ln\left|\frac{x-a}{x}\right| + C$

to integrate this separable differential equation and simplify the final arbitrary constant in your implicit solution without attempting to solve for x.

c) Now isolate x in your implicit general solution (solve for x).

d) Use your implicit general solution of part b) at $t=0$ to evaluate the arbitrary constant in terms of the initial value x_0 and backsubstitute into your solution for part c) and simplify. Does your solution agree with Maple's direct solution of the above initial value problem? [If not use Maple's solution to continue this problem!]

e) What is the value of the characteristic time τ for this exponential behavior?

f) Consider the case $x_0 = 25$ alligators at the initial time. Simplify your solution expression. What happens to this population in the long run?

g) Consider the case $x_0 = 150$ alligators at the initial time. Simplify your solution expression. What happens to this population in the long run? [Hint: "doomsday!" When does this occur?]

h) Make separate rough hand sketches of the two solutions, labeling all key points and keeping the characteristic time in mind if relevant.

a) $\dfrac{dx}{dt} = \underbrace{0.0001}_{k}x^2 - \underbrace{0.01}_{kM}x = \underbrace{0.0001\,x(x-100)}_{\text{opp sign from logistic DE}} = 0 \rightarrow x = 100 = M$

separate and integrate:

b) $\displaystyle\int \frac{dx}{x(x-100)} = \int 0.0001\,dt = 0.0001\,t + C_1$

$\displaystyle\frac{1}{100}\ln\left|\frac{x-100}{x}\right| \xrightarrow{\times 100} \ln\left|\frac{x-100}{x}\right| = 0.01t + 100C_1$

$\left|\dfrac{x-100}{x}\right| = e^{.01t + 100C_1} = e^{100C_1}e^{.01t}$

$\dfrac{x-100}{x} = \underbrace{\pm e^{100C_1}}_{\equiv C} e^{.01t}$ ← always distinguish different arbitrary constants in a given problem

$$\boxed{\frac{x-100}{x} = Ce^{.01t}}$$

c) $x - 100 = Ce^{.01t}x$

$x(1 - Ce^{.01t}) = 100$

$$\boxed{x = \frac{100}{1 - Ce^{.01t}}}$$

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d) $\dfrac{x_0-100}{x_0} = C e^{0} = C$

$$x = \frac{100}{1-\left(\frac{x_0-100}{x_0}\right)e^{.01t}} = \boxed{\frac{100\,x_0}{x_0-(x_0-100)\,e^{.01t}} = x}$$

IVP soln

Maple is a bit stupid not putting it in this simplest form but easily seen to be equivalent

$$x(t) = -\frac{100\,x_0}{e^{t/100}x_0 - 100e^{t/100} - x_0}$$

eliminate signs by multiplying thru; collect exp terms

e) $e^{t/100}$ ← $\boxed{\tau = 100}$!

f) $x_0=25$: $x = \dfrac{100\cdot 25}{25-(\underbrace{25-100}_{-75})e^{.01t}} = \dfrac{100\cdot 25}{25+75e^{.01t}} = \boxed{\dfrac{100}{1+3e^{.01t}} = x}$

$$\lim_{t\to\infty} x = \lim_{t\to\infty}\frac{100}{1+3\underbrace{e^{.01t}}_{\text{grows}}} = \lim_{t\to\infty}\frac{100}{3e^{.01t}} = 0$$

The population will go to zero, i.e., "die out."

For $t > 349.65$, $P<1$! Population is discrete. You cannot have a fractional cow

g) $x_0=150$: $x = \dfrac{100\cdot 150}{150-(\underbrace{150-100}_{50})e^{.01t}} = \dfrac{100}{1-\frac{1}{3}e^{.01t}}$

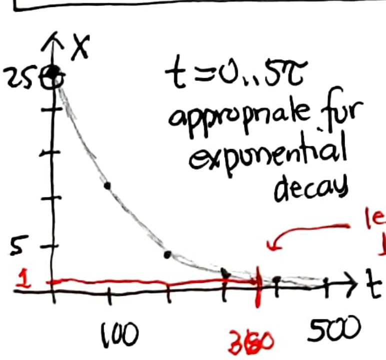
≈ 0 vertical asymptote

so this is only a math model!

$$3 = e^{.01t},\quad .01t = \ln 3,\quad t = \frac{\ln 3}{.01} = \underbrace{100}_{\tau!}\ln 3 \approx 109.861$$

The population goes infinite by 110 months, i.e., "blows up!" The model obviously breaks down near this doomsday time.

g)



$t = 0..5\tau$ appropriate for exponential decay

less than 1 cow left!