

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them, unless specifically requested. Print out this quiz to respond to the graph instructions on paper.

1. $(1+x) \frac{dy}{dx} = -y + \cos(x), y(0) = 1.$

Note the 0.2 separation in the vertical arrow grid.

a) Note the zero-slope isocline (RHS=0) curve which divides up the plane into negative slope ($y > \cos(x)$) and positive slope ($y < \cos(x)$). What is the slope at the initial point?

b) Indicate the initial data point on the graph by a circled dot annotated by an arrow pointing to the point from the initial condition written to the side of the graph and roughly draw in the corresponding solution curve, keeping in mind part a).

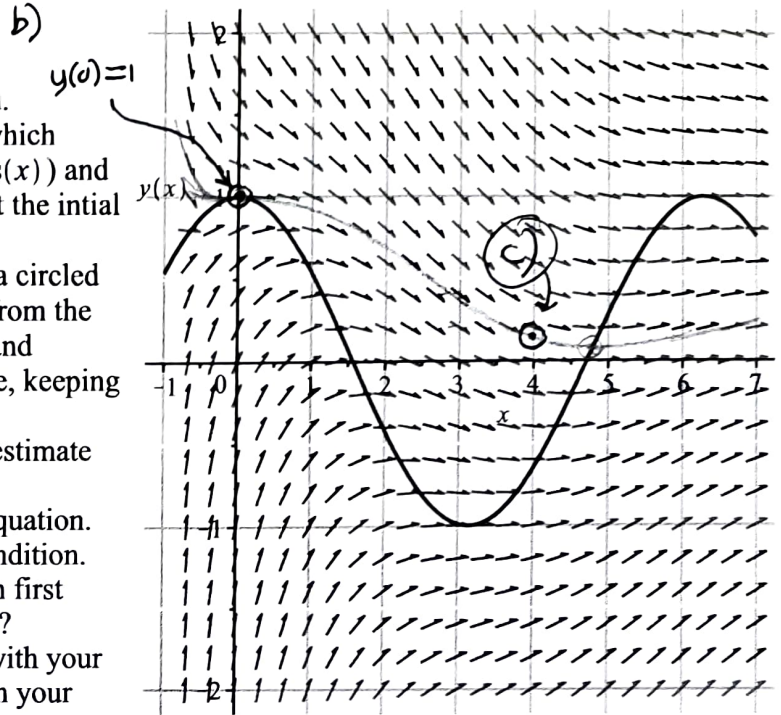
c) Put a circled dot at the point where $x=4$ and estimate the value of y there.

d) Find the general solution of this differential equation.

e) Find the solution which satisfies the initial condition.

f) At what numerical value of x does this solution first equal 0? Is this point consistent with your sketch?

g) Evaluate $y(4)$ for this solution and compare with your graphical estimate. Is this roughly consistent with your estimate? Explain if you can.



h) Check by hand that your solution to e) solves the differential equation. [Remember, backsub everywhere in the DE eliminating y , then simplify both sides independently.]

i) Enter the differential equation and the initial condition separated by a comma in Maple and solve with the context sensitive menu. Write down exactly the form of the solution that it gives you. Does it agree with your hand solution? Explain why if so. If not, you better find your error.

e) estimate: at $x=4$, my handdrawn curve is about $3/4$ of the interval between arrow centers so $(0.2) * (.75) \approx 0.15$ above $y=0$, hence

$y(4) \approx 0.15$ by graphical estimate.

(see sheet 2 for rest)

MAT 2705-04/05 20F QUIZ 3 Answer Key

a) $y'(0) = 0$ since $(0,1)$ lies on $m=0$ isocline: $\underbrace{(-1) + \cos(0)} = -1+1=0$
 RHS at $(x,y) = (0,1)$

b) see graph

c) see graph

d) $(1+x)y' + y = \cos x$

$y' + \frac{1}{1+x}y = \frac{\cos x}{1+x}$ standard form

$\int \frac{dx}{1+x} = \ln|1+x| = \ln(1+x)$ unnecessary

$(1+x) \left(y' + \frac{1}{1+x}y \right) = (1+x) \left(\frac{\cos x}{1+x} \right)$

$\frac{d}{dx}(y(1+x)) = \cos x$

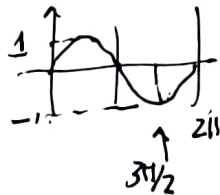
$y(1+x) = \int \cos x dx = \sin x + C$

$y = \frac{\sin x + C}{1+x}$ gen soln

e) $1 = y(0) = \frac{\sin(0) + C}{1+0} = \frac{C}{1} = C$

$y = \frac{1 + \sin x}{1+x}$ \square soln

f) $1 + \sin x = 0$
 $\sin x = -1$



$x = \frac{3\pi}{2} \approx 4.71$

My sketched curve never reaches the axis!
 It veered up too much along the way.
 It does reach its lowest pt nearby at $x \approx 4.8$
 when it crosses the isocline $m=0$.
 Not enough arrow resolution to do much better!

g) $y(4) = \frac{1 + \sin 4}{1+4} \approx 0.0486$

much less than 0.15!
 but not enough resolution to do much better.

h) $y = \frac{1 + \sin x}{1+x}$

$y' = \frac{(1+x)\cos x - (1 + \sin x)(0+1)}{(1+x)^2}$
 $= \frac{(1+x)\cos x - (1 + \sin x)}{(1+x)^2}$

$(1+x)y' = -y + \cos x$ (use original DE!)

$(1+x) \left[\frac{(1+x)\cos x - (1 + \sin x)}{(1+x)^2} \right]$
 $= -\frac{(1 + \sin x)}{1+x} + \cos x$
 $\cos x - \frac{(1 + \sin x)}{(1+x)} = \cos x - \frac{1 + \sin x}{1+x}$

i) $y(x) = \frac{\sin(x) + 1}{1+x}$

confirms hand soln.