MAT2705-04/05 20f Quiz 2 Print Name (Last, First) _____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use **proper mathematical notation**, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not subsitute for them, unless specifically requested. **Print out this quiz to respond to the graph instructions on paper.**

1. $y\frac{dy}{dx} = x(1-y^2), y(0)=2.$

Note the 0.2 separation in arrow grid.

a) Indicate the initial data point on the graph by a circled dot annotated by an arrow pointing to the point from the initial condition written to the side of the graph and roughly draw in the corresponding solution curve.

b) Put a circled dot at the point where $x = 1.2$ and estimate the value of y there.

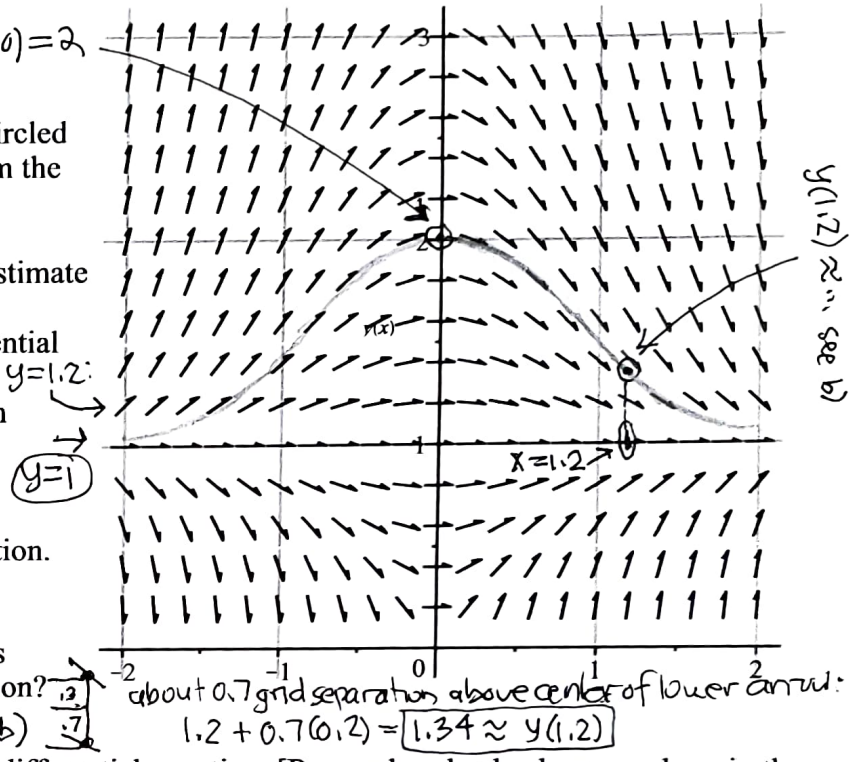
c) Find the general solution of this separable differential equation.

d) What is the obvious straight line isocline solution missing from your family unless you redefine your initially additive constant (see direction field or DE)?

e) Find the solution which satisfies the initial condition.

f) Evaluate $y(1.2)$ for this solution and mark the corresponding point on the graph by a circled dot annotated as above (arrow from $y(1.2) \approx \ldots$). Is this consistent with your approximate hand drawn solution? Compare the value with part b).

a) $y(0)=2$



b) $1.2 + 0.7(0.2) = \boxed{1.34 \approx y(1.2)}$

g) Check by hand that your solution to e) solves the differential equation. [Remember, backsub everywhere in the DE eliminating y, then simplify both sides independently.]

h) Enter the differential equation and the initial condition separated by a comma in Maple. Right click and solve. Write down exactly the form of the solution that it gives you. Does it agree with your hand solution? Explain why if so. If not, you better find your error.

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c) $\int \frac{y\,dy}{1-y^2} = \int x\,dx$ separate and integrate

$u = 1-y^2,\ du = -2y\,dy$

$-\frac{1}{2}\int \frac{du}{u} = -\frac{1}{2}\ln|u|$

$-\frac{1}{2}\ln|1-y^2| = \frac{x^2}{2} + C_1$

$\ln|1-y^2| = -x^2 - 2C_1$

$|1-y^2| = e^{-x^2-2C_1} = e^{-2C_1}e^{-x^2}$

$1-y^2 = \underbrace{\pm e^{-2C_1}}_{\equiv C_2} e^{-x^2}$

$1-y^2 = Ce^{-x^2}$

$y^2 = 1 - Ce^{-x^2}$

$\boxed{y = \pm\sqrt{1-Ce^{-x^2}}}$

$\left(\text{or } \pm\sqrt{1+Ce^{-x^2}},\ C = -(\pm e^{-2C_1})\right)$

$y>0$: + sign

$y<0$: − sign

e) $y(0)=2$: $x=0, y=2$:

$1-2^2 = Ce^0 = C \to C = 1-4 = -3$

$y = +\sqrt{1+3e^{-x^2}}$

$\boxed{y = \sqrt{1+3e^{-x^2}}}$

MAT2705 20F Quiz2 Answerkey (2)

d) $y^2=1$ makes RHS zero, derivative zero
$y=1$ or $y=-1$ are both obvious isocline solutions
but $\boxed{y=1}$ is obvious from arrows in plot

f) $y(1.2)=\sqrt{1+3e^{-1.2^2}} \approx 1.307969 \approx \boxed{1.31}$ close to estimate 1.34
too close to try to draw a new circled dot
need larger graph!

g) $y\frac{dy}{dx} = x(1-y^2)$ $\quad y=\sqrt{1+3e^{-x^2}}=(1+3e^{-x^2})^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}(1+3e^{-x^2})^{-1/2}(0+3(-e^{-x^2})(-2x))$

$= \frac{-3xe^{-x^2}}{\sqrt{1+3e^{-x^2}}}$

$\sqrt{1+3e^{-x^2}}\left(\frac{-3xe^{-x^2}}{\sqrt{1+3e^{-x^2}}}\right) = x\cdot(1-(\sqrt{1+3e^{-x^2}})^2)$

$-3xe^{-x^2} = x(1-(1+3e^{-x^2}))$

$-3xe^{-x^2} = x(-3e^{-x^2})$ ✓

only backsub and simplify both sides independantly
No even solving steps mixing sides

h) `>yy' = x_(1-y^2), y(0)= 2` → $y(x)=\sqrt{3e^{-x^2}+1}$
space space ← or asterisk for multiplication

gen soln:
Maple: $y(x)=\sqrt{e^{-x^2}\_C_1+1}, -\sqrt{e^{-x^2}\_C+1}$

bob put minus here but (−C) but if I wee not trying to go back and use a previous eqn for the initial condition, I would have renamed this