MAT2500-01/04 19S Quiz 9 (take home) Print Name (Last, First) ____________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

Consider the integral $V_{(xzy)} = \int_0^1 \int_y^1 \int_0^z 1 \, dx\, dz\, dy$.

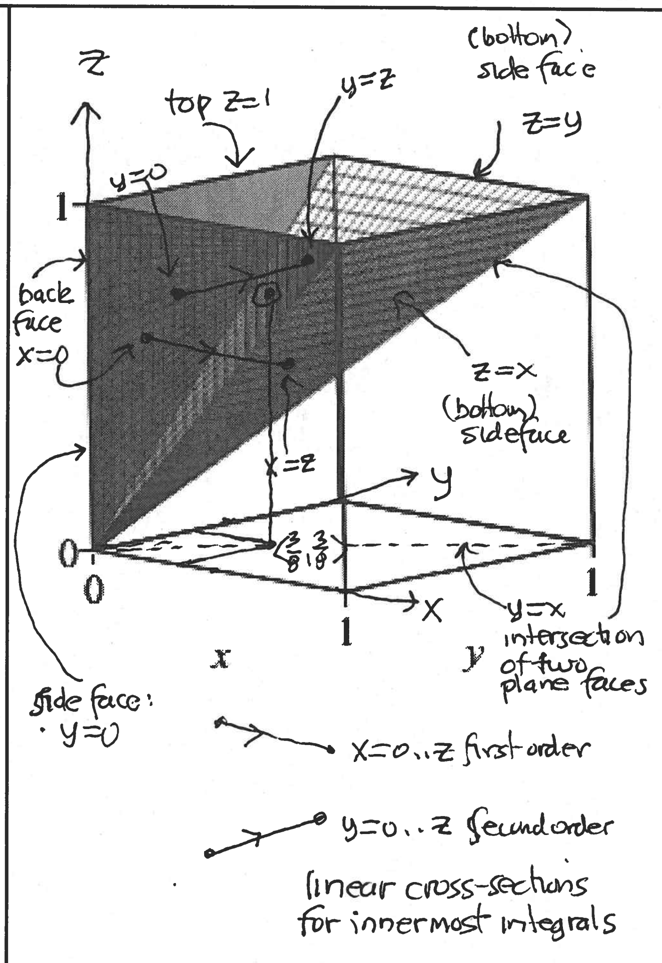
a) Deconstruct the integral by assigning variables to each of the 5 limits to obtain the equations of the faces of the solid region. Draw first a z-y plane diagram for the outer double integral, putting into it a typical linear cross-section indicating the z partial integration in it labeled by the starting and stopping values of z, with an arrow midway to indicate increasing z. The draw the x-z plane diagram needed for the innermost integration step. Convince yourself that together they correspond to the diagram at the right. Make an attempt to draw them together. Put in a fully labeled linear cross section in the diagram to the right for the innermost partial integration.

b) Then rewrite the iteration in the order $V_{(yxz)}$, and justify with an fully labeled diagram indicating the outer double integral, and include a fully labeled linear cross section in the diagram to the right for the innermost partial integration.

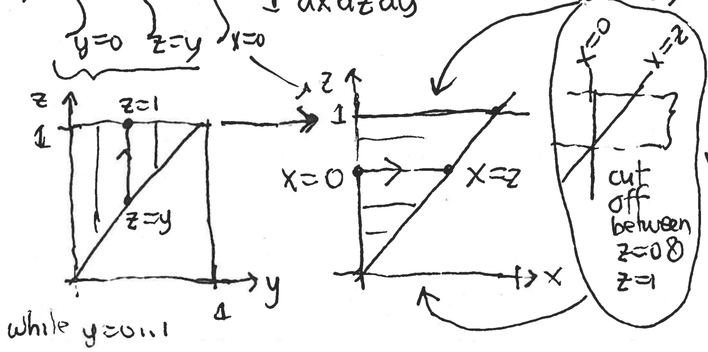
c) Evaluate these integrals exactly with Maple (they should agree).

d) Pick the original order and use Maple to evaluate the 3 volume moments about the coordinate planes
$\langle Vyz, Vxz, Vxy \rangle = \iiint \langle x, y, z \rangle \, dV$ and finally the centroid
$\langle \bar{x}, \bar{y}, \bar{z} \rangle = \dfrac{\langle Vyz, Vxz, Vxy \rangle}{V}$ exactly and numerically.

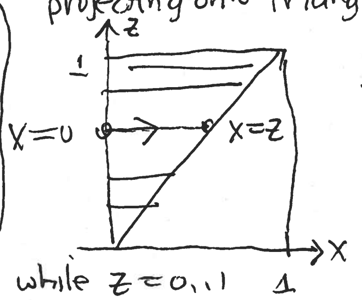
e) Check by redoing this with your new iteration.



a) $\int_{y=0}^{y=1} \int_{z=y}^{z=1} \int_{x=0}^{x=z} 1 \, dx\,dz\,dy$



b) y first means starting at y=0 and moving to y=z projecting onto triangle face on x-z plane



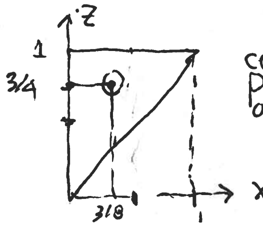
$$\boxed{\int_0^1 \int_0^z \int_0^z 1 \, dy\,dx\,dz}$$

c) $\mathrm{Int}(a) = \mathrm{Int}(b) = \boxed{1/3}$

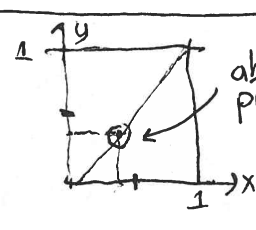
$\div \frac{1}{3}$ means $\times 3$

d) $\int_0^1 \int_y^1 \int_0^z \langle x, y, z\rangle \, dx\,dz\,dy = \langle \frac{1}{8}, \frac{1}{8}, \frac{1}{4} \rangle \rightarrow \boxed{\langle \bar{x}, \bar{y}, \bar{z} \rangle = \langle \frac{3}{8}, \frac{3}{8}, \frac{3}{4} \rangle = \langle .375, .375, .75 \rangle}$

optional: makes senses



centroid projection onto vertical faces



above this point in base projection