MAT2500-01/04 19S Quiz 8 Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

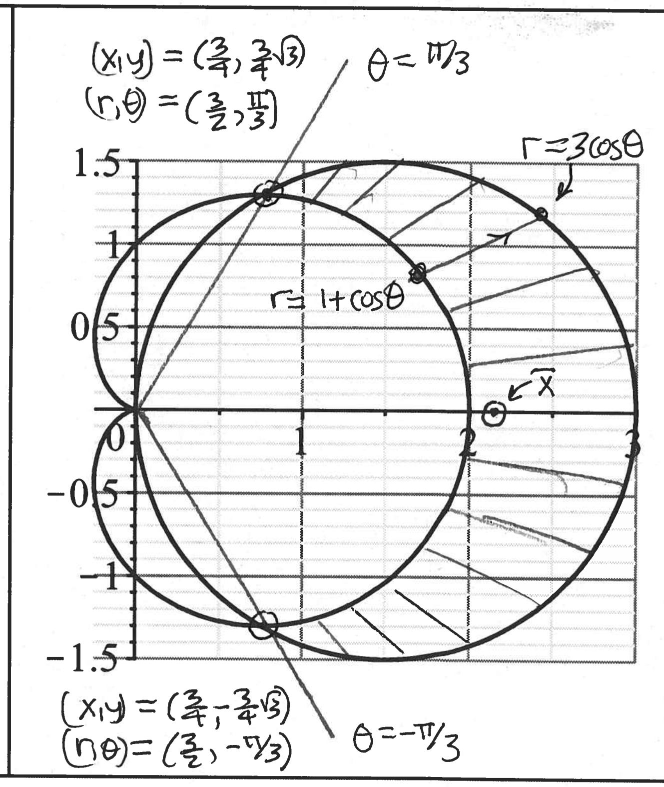
a) Find the points of intersection of the two curves away from the origin:
$r=1+\cos(\theta)$, $r=3\cos(\theta)$ shown in the figure and label them with both the Cartesian coords $(x, y)=$... and the polar coordinates $(r, \theta)=$...

b) "Shade" the region outside the cardiod but inside the circle with equally spaced radial cross-sections and label a typical one by bullet endpoints (and arrowhead at the midpoint) with the starting and stopping values of r as equations, and indicate the angular range by half rays at the starting and stopping values of the polar angle labeled by those equations.

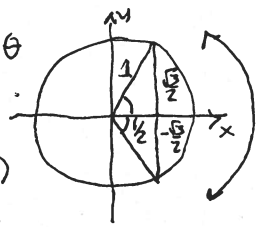
c) Set up an iterated integral for the area of this region, and use Maple to evaluate it exactly and approximately.

d) Set up an iterated integral for the integral of x over this region, and evaluate it in Maple, exactly and approximately.

e) By symmetry the centroid of this region lies on the horizontal axis at the average value of x. Evaluate this approximately to 2 decimal places and then mark it clearly in the diagram. Can you put into words why it seems to be in the correct location?



a) $1+\cos\theta = 3\cos\theta$
$1 = 2\cos\theta$
$\cos\theta = \frac{1}{2}$
$\theta = \pm\pi/3$ (60°)



need continuous range of values: $-\frac{\pi}{3} \le \theta \le \frac{\pi}{3}$
so $\theta = 2\pi - \frac{\pi}{3}$ not useful for lower point

$r = 3\cos\pi/3 = 3/2$
$x = \frac{3}{2}\cos\pi/3 = \frac{3}{2}(\frac{1}{2}) = 3/4$
$y = \frac{3}{2}\sin(\pm\pi/3) = \pm\frac{3}{2}(\frac{\sqrt{3}}{2}) = \pm\frac{3\sqrt{3}}{4}$

b) see plot

c) $A = \iint_R 1\, dA = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \int_{1+\cos\theta}^{3\cos\theta} 1\, r\, dr\, d\theta \overset{\text{Maple}}{=} \boxed{\pi \approx 3.14159}$

$\left(= 2\int_0^{\pi/3}\int_{1+\cos\theta}^{3\cos\theta} 1\, r\, dr\, d\theta\right)$

d) $A_x \equiv \iint_R x\, dA = \int_{-\pi/3}^{\pi/3}\int_{1+\cos\theta}^{3\cos\theta} (r\cos\theta)\, r\, dr\, d\theta \overset{\text{Maple}}{=} \boxed{\frac{9\sqrt{3}}{16} + \frac{11\pi}{6} \approx 6.73387}$

e) $\bar{x} = x_{avg} = \frac{A_x}{A} = \frac{\frac{9\sqrt{3}}{16} + \frac{11\pi}{6}}{\pi} = \frac{9\sqrt{3}}{6\pi} + \frac{11}{6} \overset{\text{Maple}}{\approx} 2.1434 \approx \boxed{2.14}$.

Notice the vertical line $x = 2$. Most of the area is to the right of this line so the centroid must be to the right. Clearly it must also be left of the vertical line $x = 2.5$, in fact much closer to $x=2$ than $x = 2.5$.