MAT2500-01/04 18S Quiz 6   Print Name (Last, First) ______________________________|___

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

1. Consider the level surface

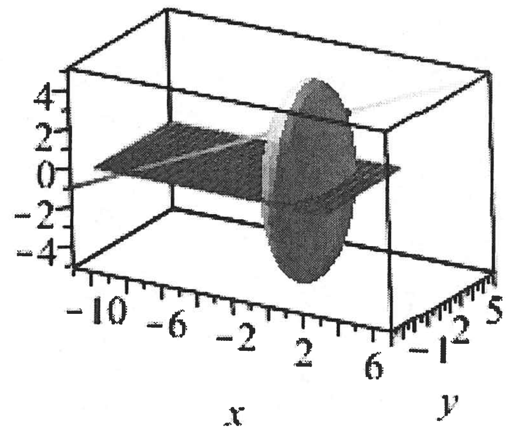
$F(x, y, z) = x^2 + \frac{y^2}{4} + \frac{z^2}{9} = 3$ at the point (1,2,3).

3 a) Write the equation for the tangent plane simplifying it to the standard linear form, then solving for z.

2 b) Write a parametrized vector equation for the normal line at the point (1,2,3) .

2 c) Where does the normal line at the point (1,2,3) on this surface intersect the plane $z = 0$?

3 d) Evaluate the directional derivative of F at (1,2,3) in the direction towards the origin, and approximate it to 3 decimal places.



▶ solution

1 a) $\vec{\nabla} F(x,y,z) = \langle 2x, \frac{2y}{4}, \frac{2z}{9} \rangle = \langle 2x, \frac{y}{2}, \frac{2z}{9} \rangle$

$\vec{\nabla} F(1,2,3) = \langle 2(1), \frac{2}{2}, \frac{2(3)}{9} \rangle = \langle 2, 1, 2/3 \rangle \xrightarrow{\times 3} \vec{n} = \langle 6, 3, 2 \rangle$

$\vec{r}_0 = \langle 1,2,3 \rangle$, $0 = \vec{n} \cdot (\vec{r} - \vec{r}_0) = \langle 6,3,2 \rangle \cdot \langle x-1, y-2, z-3 \rangle = 6(x-1) + 3(y-2) + 2(z-3)$

$= 6x + 3y + 2z - (6+6+6) = 6x + 3y + 2z - 18$

$\boxed{6x + 3y + 2z = 18} \rightarrow \boxed{z = \frac{1}{2}(18 - 6x - 3y) = 9 - 3x - \frac{3}{2}y}$

b) $\vec{r} = \vec{r}_0 + t\vec{n} = \langle 1,2,3 \rangle + t\langle 6,3,2 \rangle = \langle 1+6t, 2+3t, 3+2t \rangle$

* $\boxed{\langle x, y, z \rangle = \langle 1+6t, 2+3t, 3+2t \rangle}$ (you could have just used $\vec{\nabla} F(1,2,3) = \vec{n}$)

c) $0 = z = 3 + 2t \rightarrow t = -3/2 \rightarrow x = 1 + 6(-3/2) = 1 - 9 = -8$

$y = 2 + 3(-3/2) = 2 - \frac{9}{2} = -\frac{5}{2}$ $\boxed{\text{point: } (-8, -\frac{5}{2}, 0)}$

d) $\vec{PO} = \langle 0,0,0 \rangle - \langle 1,2,3 \rangle = -\langle 1,2,3 \rangle$ $\hat{PQ} = -\frac{\langle 1,2,3 \rangle}{\sqrt{1+4+9}} = -\frac{\langle 1,2,3 \rangle}{\sqrt{14}}$

identify what you are calculating!

$D_{\hat{PQ}} F(1,2,3) = \hat{PQ} \cdot \vec{\nabla} F(1,2,3) = -\frac{\langle 1,2,3 \rangle}{\sqrt{14}} \cdot \langle 2, 1, 2/3 \rangle = -\frac{1}{\sqrt{14}}(2 + 2 + 2) = \boxed{-\frac{6}{\sqrt{14}}}$

≈ -1.60356

≈ -1.604

* be explicit, replace $\vec{r}$ by $\langle x,y,z \rangle$ since $\vec{r}$ is not universally understood

also "normal line = ..." or "$N(1,2,3) = \ldots$" fail to give x,y,z as functions of t!