MAT2500-01/04 19s Quiz 4 Take Home Print Name (Last, First) _____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation if appropriate). Indicate where technology is used and what type (Maple, GC).

1. The wind-chill index W is the perceived temperature when the actual temperature is T (both in degrees Centigrade) and the wind speed is v (in km/s), so we can write $W = f(T, v)$. Values are recorded in the table.

(a) What is the value of $f(-15, 30)$? What is its meaning in words (a complete sentence)?
(b) For what value of v is $f(-10, v) = -23$? Formulate this question in words.
(c) What is the meaning of the function $W = f(-15, v)$?

Wind speed (km/h)

Actual temperature (°C) T \ v	20	30	40	50	60	70
−10	−18	−20	−21	−22	−23	−23
−15	−24	−26	−27	−29	−30	−30
−20	−30	−33	−34	−35	−36	−37
−25	−37	−39	−41	−42	−43	−44

(d) Evaluate the average rate of change for $f(-15, v)$ for the intervals $v = 20..30$ and then $v = 30..40$ and then average these to get a decimal value for the "instantaneous" rate of change of $f(-15,v)$ at $v = 30$: $\frac{d}{dv} f(-15, v) \Big|_{v=30}$.

e) Using this result, to what value would you expect the perceived temperature to increase to if the wind speed decreases from 30 km/h to 28 km/h an actual temperature of -15 degrees?

▶ solution

a) $f(-15, 30) = -26$. When the actual temperature is $-15°C$ and the wind speed is 30 km/h, the perceived temperature is $-26°C$.

(b) $f(-10, 50) = -23$ so $v = 50$. At what wind speed is the perceived temperature $-23°C$ when the actual temperature is $-10°C$.

(c) When the actual temperature is $-15°C$, this function gives the perceived temperature as a function of the windspeed in km/h.

(d)

	20	30	40
−15	−24	−26	−27

$\Delta v = 10$

$\Delta W = -26 - (-24) = -2$ $\quad \Delta W = -27 - (-26) = -1$

$\frac{\Delta W}{\Delta v} = -\frac{2}{10}$ $\quad \frac{\Delta W}{\Delta v} = -\frac{1}{10}$

$\left(\frac{\Delta W}{\Delta v}\right)_{avg} = \frac{1}{2}(-.2 - .1) = -0.15 = \frac{dW}{dv}\Big|_{\substack{T=-15 \\ v=30}}$

e) Per unit increase in the speed, this derivative gives the change in the value of the function in the linear approximation.

$\Delta v = 28 - 30 = -2$

$\Delta W = (-0.15)(-2) = 0.3$

$W + \Delta W = -26 + 0.3 = \boxed{-25.7°C}$

The perceived temperature would rise to this slightly higher value.