MAT2500-01/04 19S Test 2 Answers

① a) $f(xyz) = ze^{xy}$

$f_x(xyz) = yze^{xy}$

$f_y(xyz) = xze^{xy}$

$f_z(xyz) = e^{xy}$

$\vec{\nabla} f(x,y,z) = e^{xy}\langle yz, xz, 1\rangle$

$\vec{\nabla} f(0,1,2) = e^0\langle 1(2), 0(2), 1\rangle$

$= \langle 2,0,1\rangle$

$|\vec{\nabla} f(0,1,2)| = \sqrt{2^2+0^2+1^2} = \sqrt{5}$

$\hat{\nabla} f(0,1,2) = \boxed{\frac{1}{\sqrt{5}}\langle 2,0,1\rangle = \hat{u}}$

b) max rate increase:

$|D_{\hat{u}} f(0,1,2)| = |\vec{\nabla} f(0,1,2)| = \boxed{\sqrt{5}}$

c) $\vec{PO} = \langle 0,0,0\rangle - \langle 0,1,2\rangle = -\langle 0,1,2\rangle$

$\hat{PO} = -\frac{\langle 0,1,2\rangle}{\sqrt{5}}$

$D_{\hat{PO}} f(0,1,2) = \hat{PO}\cdot\vec{\nabla} f(0,1,2)$

$= -\frac{1}{\sqrt{5}}\langle 0,1,2\rangle\cdot\langle 2,0,1\rangle = \boxed{-\frac{2}{\sqrt{5}}}$

d) $\vec{r}(t) = \langle \underbrace{2(t-1)}_{=0\to t=1}, t^2, 2t^3\rangle \to \vec{r}(1) = \langle 0,1,2\rangle$ ✓

$\vec{r}\,'(t) = \langle 2, 2t, 6t^2\rangle$

$\vec{r}\,'(1) = \langle 2,2,6\rangle = 2\langle 1,1,3\rangle$

$\frac{d}{dt} f(x(t),y(t),z(t)) = \frac{\partial f}{\partial x}(\vec{r}(t))\frac{dx(t)}{dt} + \frac{\partial f}{\partial y}(\vec{r}(t))\frac{dy(t)}{dt} + \frac{\partial f}{\partial z}(\vec{r}(t))\frac{dz(t)}{dt}$

$= \vec{r}\,'(t)\cdot\vec{\nabla} f(\vec{r}(t))$

$\frac{df(\vec{r}(t))}{dt}\Big|_{t=1} = \vec{r}\,'(1)\cdot\vec{\nabla} f(1,0,2)$

$= 2\langle 1,1,3\rangle\cdot\langle 2,0,1\rangle = 2(2+0+3) = \boxed{10}$

e) $f(0,1,2) = 2e^0 = 2 \to \boxed{ze^{xy} = 2}$ level surface thru $(0,1,2)$

$\vec{\nabla} f(0,1,2) = \langle 2,0,1\rangle = \vec{n}$ normal to tangent plane thru $\vec{r}_0 = \langle 0,1,2\rangle$

$0 = \vec{n}\cdot(\vec{r}-\vec{r}_0) = \langle 2,0,1\rangle\cdot\langle x-0, y-1, z-2\rangle$

$= 2x+z-2 \to \boxed{2x+z=2}$

$L(x,y,z) = f(0,1,2) + \vec{\nabla} f(0,1,2)\cdot(\vec{r}-\langle 0,1,2\rangle)$

$= 2 + \langle 2,0,1\rangle\cdot\langle x-0, y-1, z-2\rangle$

$= 2 + 2x + z - 2 = \boxed{2x+z = L(xyz)}$

② $V = \frac{2\pi}{3}abc \qquad (a,b,c) = (185, 110, 38)$

$dV = \frac{2\pi}{3}(bc\,da + ac\,db + ab\,dc)$

$da = db = dc = \frac{1}{8}\left(\frac{1}{12}\right) = \frac{1}{96}$

$dV = \frac{2\pi}{3}(bc+ac+ab)\frac{1}{96} = \frac{\pi}{3(48)}(bc+ac+ab)$

$dV\big|_{(a,b,c)=(185,110,38)} = \frac{\pi}{3(48)}(110(38)+185(38)+185(110))$

$= \frac{14855\pi}{72} \approx 648.172 \approx \boxed{648\ ft^3}$

③ $\underbrace{\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}}_{\equiv F(x,y,z)} = 1 \leftarrow$ level surface of F

$\vec{\nabla} F(x,y,z) = \langle \frac{2x}{a^2}, \frac{2y}{b^2}, \frac{2z}{c^2}\rangle$

$= 2\langle \frac{x}{a^2}, \frac{y}{b^2}, \frac{z}{c^2}\rangle$

$\vec{\nabla} F(x_0,y_0,z_0) = 2\underbrace{\langle \frac{x_0}{a^2}, \frac{y_0}{b^2}, \frac{z_0}{c^2}\rangle}_{\vec{n}_0}$ normal to level surface

$\vec{r}_0 = \langle 100, 50, 26.9\rangle$

$= \langle x_0, y_0, z_0\rangle$

$\boxed{\langle x,y,z\rangle = \vec{r} = \vec{r}_0 + t\vec{n}_0} = \langle x_0 + \frac{tx_0}{a^2}, y_0+\frac{ty_0}{b^2}, z_0+\frac{tz_0}{c^2}\rangle$

$\boxed{= \langle \underbrace{x_0(1+\tfrac{t}{a^2})}_{x_s}, \underbrace{y_0(1+\tfrac{t}{b^2})}_{y_s}, \underbrace{z_0(1+\tfrac{t}{c^2})}_{=0\to t=-c^2}\rangle}$

$x_s = x_0(1-\frac{c^2}{a^2})$, $y_s = y_0(1-\frac{c^2}{b^2})$

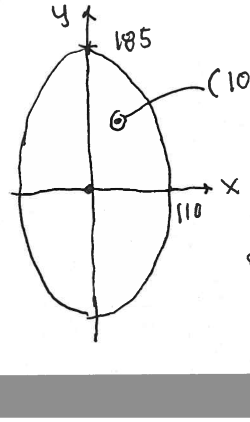
$= 100(1-(\frac{38}{185})^2)$, $= 50(1-(\frac{38}{110})^2)$

$\approx 95.78$ $\approx 44.03$

$\approx 95.8$ $\approx 44.0$

so $\boxed{(x_s,y_s) \approx (95.8, 44.0)}$

both slightly towards center (0,0) from (100,50) as one would expect from the tilt away from the vertical by the roof.




dome floor region