MAT2500-01/04 19S Quiz 0. Print Name (Last, First) _____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

a) Find the center C and radius of this sphere: $x^2+y^2+z^2+2x-6y+8z+17=0$.
[Hint: check with Load Package Student Precalculus]

b) Find the distance s_1 between that center and the point $P(2, 6, -7)$.

c) What is the distance s_2 from that point P to the nearest point on the sphere?

d) What percent of the first distance s_1 is the second distance s_2?

e) Take your best shot at drawing a 2-dimensional diagram corresponding to any plane containing the center and the given point, clearly labeling the radius and the two separation distances in your diagram.

f) Is the value of d) consistent with your diagram? Explain.

▶ solution

a) $-17 = x^2+2x \;=\; (x+1)^2-1$
$+y^2-6y \;+\; (y-3)^2-9$
$+z^2+8z \;+\; (z+4)^2-16$
$-17-9$

$(x+1)^2+(y-3)^2+(z+4)^2 = 9 = 3^2$

$\boxed{C: (-1, 3, -4),\ r=3}$

b) $s_1 = \sqrt{(2+1)^2+(6-3)^2+(-7+4)^2}$
$= \sqrt{3^2+3^2+3^2} = \boxed{3\sqrt{3}} \approx 5.20$

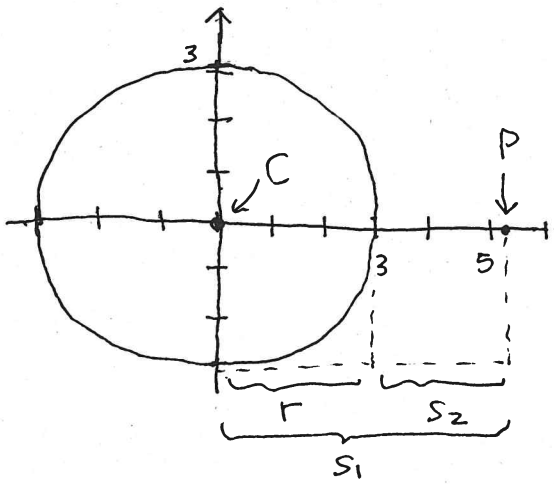
c) $s_2 = s_1 - r = \boxed{3\sqrt{3}-3} = 3(\sqrt{3}-1)$
≈ 2.20

← subtracting the radius from the line segment CP gives the remaining distance

d) $\frac{s_2}{s_1} = \frac{3(\sqrt{3}-1)}{3\sqrt{3}} = 1-\frac{1}{\sqrt{3}} \approx 0.4226$
≈ 0.42

$\boxed{\text{about } 42\%}$ only needed for comparison with figure so high precision not warranted

e)



f) s_2 is a bit less than half of s_1 so 42% is in the right ballpark