MAT1505-03/04 19F Quiz 8 Print Name (Last, First) ________________, ________________|____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

No technology is needed today. Use words to explain, show appropriate algebra and limits or limiting behavior.

1. Is $\sum_{n=1}^{\infty} \frac{n^{100}\,100^n}{n!}$ **convergent? Justify your claim.**

2. Is $\sum_{n=1}^{\infty} (-1)^{n-1}\frac{\sqrt{n}}{1+n^2}$ **absolutely convergent, conditionally convergent or divergent? Justify your claim.**

3. Is $\sum_{n=1}^{\infty} (-1)^{n-1}\frac{\sqrt{n}}{1+n}$ **absolutely convergent, conditionally convergent or divergent? Justify your claim.**

▶ solution

① $\frac{a_{n+1}}{a_n} = \frac{\frac{(n+1)^{100}\,100^{n+1}}{(n+1)!}}{\frac{n^{100}\,100^n}{n!}} = \frac{(n+1)^{100}}{n^{100}}\,\frac{100^{n+1}}{100^n}\,\frac{n!}{(n+1)!} = \left(\frac{n+1}{n}\right)^{100} \frac{100^n\cdot 100}{100^n}\,\frac{n!}{(n+1)\,n!}$

$= \Big(1+\underbrace{\tfrac{1}{n}}_{\to 0}\Big)^{100}\cdot 100\cdot \Big(\underbrace{\tfrac{1}{n+1}}_{\to 0}\Big) \xrightarrow{\text{large } n} \frac{100}{n+1} \xrightarrow{n\to\infty} 0$

The limiting ratio of successive terms is zero, so this $\boxed{\text{converges}}$ (faster than any geometric series, no matter how small the positive ratio).

② $\left|\frac{a_{n+1}}{\cancel{a_n}}\right| = \frac{n^{1/2}}{1+n^2} \xrightarrow{\text{large } n} \frac{n^{1/2}}{n^2} = \frac{1}{n^{3/2}}$ so this has the same behavior as the alternating $p=3/2$ series whose absolute value series converges since $p>1$ so it $\boxed{\text{converges absolutely}}$.

③ $|a_{n+1}| = \frac{n^{1/2}}{1+n} \xrightarrow{\text{large } n} \frac{n^{1/2}}{n} = \frac{1}{n^{1/2}}$ so this has the same behavior as the alternating $p=1/2$ series whose absolute value series diverges because $p<1$ but it converges since the absolute value series decreases monotonically to zero (at least after the first few terms, but those dont matter), so this $\boxed{\text{converges conditionally.}}$