MAT1505-03/04 19F Quiz 6 Print Name (Last, First) ________________, ________________|____
Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC).

[see reverse for diagrams]

1. a) Find the arclength of the graph of the function $y = 2\ln\left(\sin\left(\frac{x}{2}\right)\right)$ on the interval $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$. Note that the integrand simplifies because it involves a perfect square, which can be re-expressed in terms of the csc function. Use Maple to find its antiderivative and finish the evaluation of the integral by hand.
b) Compare your result with the $n = 2$ secant line approximation shown in the figure. What is that approximate value numerically and what is the percentage increase in the actual length?

2. The upper half of the unit circle $x^2 + y^2 = 1$ is the graph of the function $f(x) = \sqrt{1 - x^2}$.
a) What is the arclength of the portion of this circle in the first quadrant, based on precalculus mathematics knowledge?
b) Set up and simplify the integrand in the integral representing this arclength in the first quadrant (be sure to order the lower and upper limits so that the result is positive), then use Maple to find an antiderivative, and use that antiderivative to evaluate the integral by hand.
c) Note that from the diagram that the angle $\theta = \arccos(x) = s(x)$ equals the (positive) arclength of the corresponding arc of the circle. Set up an integral for this arclength function $s(x) = \int_x^1 \sqrt{\ldots}\; dt$ giving the (positive) arclength of the arc of the circle above the interval $[x, 1]$ on the x-axis, using t for the dummy variable in the integral. Use Maple's antiderivative to evaluate this integral. Do you see why Maple's result coincides with what we already know to be this arclength function? [See diagram on page 2.]
d) Evaluate your arclength function at $x = 0$ to confirm the quarter circle result of parts a),b).

1 a) $y = 2\ln\sin\frac{x}{2}$

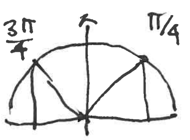
$\frac{dy}{dx} = \frac{2}{\sin\frac{x}{2}}\frac{d}{dx}\left(\sin\frac{x}{2}\right) = \frac{2}{\sin\frac{x}{2}}\cos\frac{x}{2}\cdot\frac{1}{2} = \frac{\cos\frac{x}{2}}{\sin x/2} = \cot\frac{x}{2}$

$1+\left(\frac{dy}{dx}\right)^2 = 1+\cot^2\frac{x}{2} = 1+\frac{\cos^2\frac{x}{2}}{\sin^2\frac{x}{2}} = \frac{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}}{\sin^2\frac{x}{2}} = \csc^2\frac{x}{2}$

$\sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{\csc^2\frac{x}{2}} = \csc\frac{x}{2}$

$L = \int_{\pi/2}^{3\pi/2} \csc\frac{x}{2}\,dx \overset{\text{Maple}}{=} -2\ln\left|\csc\frac{x}{2}+\cot\frac{x}{2}\right|\Big|_{\pi/2}^{3\pi/2}$

$= -2\ln\left|\underbrace{\csc\frac{3\pi}{4}}_{\sqrt{2}}+\underbrace{\cot\frac{3\pi}{4}}_{-1}\right| + 2\ln\left|\underbrace{\csc\frac{\pi}{4}}_{\sqrt{2}}+\underbrace{\cot\frac{\pi}{4}}_{1}\right|$



$= \boxed{2\ln(\sqrt{2}+1) - 2\ln(\sqrt{2}-1)}$

$= \boxed{2\ln\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right) \approx 3.5255}$

b) $y\left(\frac{\pi}{2}\right) = 2\ln\sin\left(\frac{\pi}{4}\right) = 2\ln\frac{1}{\sqrt{2}} = -2\ln\sqrt{2}$

$L_2 = 2\sqrt{(2\ln\sqrt{2})^2+(\pi/2)^2} \approx 3.4339$

$L/L_2 \approx 1.02668$

$\boxed{\text{L is about 2.6\% bigger than } L_2}$

2 a) arclength on the unit circle is the radian angle, so the quartercircle has arclength $\boxed{\pi/2}$

b) $y = \sqrt{1-x^2} = (1-x^2)^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}(1-x^2)^{-1/2}(-2x) = \frac{-x}{\sqrt{1-x^2}}$

$1+\left[\frac{dy}{dx}\right]^2 = 1+\frac{x^2}{1-x^2} = \frac{1-x^2+x^2}{1-x^2} = \frac{1}{1-x^2}$

$\sqrt{1+\left(\frac{dy}{dx}\right)^2} = \frac{1}{\sqrt{1-x^2}}$

$L = \int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx \overset{\text{Maple}}{=} \arcsin x\Big|_0^1$

$= \arcsin 1 - \arcsin 0 = \pi/2 - 0 = \boxed{\pi/2}$

c) $s(x) = \int_x^1 \frac{1}{\sqrt{1-t^2}}\,dt = \arcsin t\Big|_x^1$

$= \arcsin 1 - \arcsin x = \boxed{\frac{\pi}{2} - \arcsin x}$

trig identity: $\hookrightarrow = \arccos x$!

d) $s(0) = \frac{\pi}{2} - \arcsin 0 = \pi/2$ ✓ checks!